UNM Physics 262, Fall 2006
SAMPLE Midterm Exam 2: Relativity

Name and/or CPS number: _____

Show all your work for full credit. Remember that quantities have units and vectors have components (or magnitude and direction). **ASK** if anything seems unclear.

CALCULATORS AND CELL PHONES ARE PROHIBITED.
USE OF THESE WILL RESULT IN A ZERO FOR THE EXAM.
Keep any factors of π, e, $\sqrt{2}$, etc. in your answers.

You may use a single 8.5" $\times$ 11" paper containing notes you have prepared ahead of time to assist you.

Apportion your time sensibly. Spend about 10–12 minutes per problem.

Please put a box around your final answers.

Problem 1: _____

Problem 2: _____

Problem 3: _____

Problem 4: _____

1. Short answer [25 points]

The questions below should be answered with no more than five lines of text and no calculations. Please be brief and to the point.

[5] *a*) Define an inertial reference frame. Give an example of a *non*-inertial reference frame.

[5] *b*) What does it mean for two spacetime events to be spacelike separated? Timelike separated? Lightlike separated?

[5] *c*) In words, what are the important differences between the Doppler shift derived using special relativity and the Doppler shift derived using Galilean relativity?

[5] *d*) Evaluate the following statement: "As the speed of a moving clock approaches the speed of light, time stops."

[5] *e*) State the postulates upon which Einstein based special relativity.

2. Spacetime geometry [25 points]

By chance, the twins Alice and Bob have enrolled in Physics 262. Inspired by the course, Bob has devised a clever strategy for preserving his youth. (Alice is more sensible.) Bob has attached himself to a spring so that his x-coordinate oscillates according to

$$x(t) = x_0 \sin \omega t.$$

The product $x_0\omega$ has units of velocity, so define $\beta_0 \equiv x_0\omega/c$. Excited by the prospects of preserving his youth, but wary of testing the limits of his mechanics, Bob sets up the spring so that $\beta_0 \ll 1$, and begins to oscillate. Alice watches, rolling her eyes, from her inertial reference frame.

[8] *a*) Write down an expression relating the differential passage of time in Alice's IRF (dt) to the differential passage of Bob's proper time ($d\tau$). Simplify the relation to the point where β_0 is the only variable in the expression.

[9] *b*) Integrate this expression to find how much younger Bob is than Alice *after each oscillation*, that is, each time Bob returns to $x = 0$. [Hint: Use $\beta_0 \ll 1$ and the binomial expansion to make the calculation possible.]

[8] *c*) For $\beta_0 = 1/4$, how long would Bob have to oscillate to be one year younger than Alice?

3. Relativistic kinematics [25 points]

In the distant future, Renaissance festivals have not gone away but instead have become more extreme. Two future "knights" in such a festival will engage in a joust. To ensure fairness, each rides an identical robotic horse that can travel at a maximum speed of $c/3$ relative to the Earth. Each knight is also equipped with an identical 1 m long lance. The favored knight goes by the moniker "The Green Knight," because he wears a stylized green costume ($\lambda = 500$ nm). The challenger knight wears no special decoration. The two knights approach one another at maximum speed with their lances directed towards one another.

	Red	Orange	Yellow	Green	Blue	Indigo	Violet
λ (nm)	650	600	575	500	475	450	400

[6] *a*) At what speed does the challenger measure the Green Knight to be approaching?

[6] *b*) What length do the spectators (at rest with respect to the ground) measure the Green Knight's lance to be?

[6] *c*) What length does the challenger measure the Green Knight's lance to be?

[6] *d*) What color does the challenger measure the Green Knight's costume to be? (Use the table above to convert wavelength to color.)

[1] *e*) Would the spectators (who see the knights at a side view) measure the Green Knight's costume to be redshifted, blueshifted, or unshifted?

4. Relativistic Dynamics [25 points]

A particle is measured in a certain inertial reference frame to have a total energy of 5 GeV and a momentum of 3 GeV/c (*i.e.*, cp, which has the dimensions of energy, is equal to 3 GeV).

[6] *a*) What is the mass of the particle, in GeV/c^2?

[6] *b*) What is the velocity of the particle, v/c?

[6] *c*) What is the energy of the particle in an IRF in which the particle's momentum is 4 GeV/c?

[6] *d*) What is the kinetic energy of the particle in this new IRF?

[1] *e*) What is the maximum momentum this particle can have, according to the limits set by special relativity?