UNM Physics 262, Fall 2006
SAMPLE Midterm Exam 3: Quantum Mechanics

Name and/or CPS number: ______________________________

Show all your work for full credit. Remember that quantities have units and vectors have components (or magnitude and direction). **ASK** if anything seems unclear.

CALCULATORS AND CELL PHONES ARE PROHIBITED.
USE OF THESE WILL RESULT IN A ZERO FOR THE EXAM.
Keep any factors of π, e, $\sqrt{2}$, etc. in your answers.

You may use a single 8.5" × 11" paper containing notes you have prepared ahead of time to assist you.

Apportion your time sensibly. Spend about 10–12 minutes per problem.

$\boxed{\text{Please put a box around your final answers.}}$

Useful constants:

$\hbar$ $(= h/2\pi)$: Planck's constant.
e: The charge of an electron.
$hc = 1240$ eV·nm
c: The speed of light.
ϵ_0: The permittivity of free space.

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

1. Short answer [25 points]

[5] *a*) **de Broglie hypothesis.** To image a virus of linear size d, through what voltage must electrons (of mass m and charge e) in an electron microscope be accelerated? Assume that the electrons' motion is nonrelativistic and express your answer in terms of d, m, e and $\hbar$.

[4] *b*) **Blackbody radiation.** Planck's blackbody radiation law states that the spectral emittance of a blackbody depends on its wavelength as

$$I(\lambda) = \frac{4\pi^2\hbar c^2}{\lambda^5\left(e^{2\pi\hbar c/\lambda k_B T} - 1\right)}.$$

i) What are the units of spectral emittance, in the SI system?

ii) Wien's Law states that, at very small wavelengths, the spectral emittance of a blackbody is

$$I(\lambda) = \frac{A}{\lambda^5} e^{-B/\lambda T},$$

where A and B are constants. What are A and B in terms of the more fundamental constants $\hbar$, c, and k_B?

[8] *c*) **Compton scattering.** A photon strikes an electron at rest and scatters from it elastically. If the photon ends with a wavelength equal to three times the Compton wavelength and loses half its energy in the collision, at what angle relative to its initial propagation direction is the photon scattered? (Express this angle in radians, between 0 and π.)

[8] *d*) **Photoelectric effect.** Light of wavelength 50 nm strikes a clean metal surface in vacuum, emitting electrons of maximum kinetic energy 12.4 eV. To emit electrons with twice this kinetic energy, what wavelength light should be used, in nm? (Use $hc = 1240$ eV·nm to do your calculation.)

2. Bohr quantization [25 points]

Consider an electron (mass m_e, charge $-e$) in the $n = 2$ energy level of an atom having atomic number Z. It turns out that for $Z > 2$, there are always 2 electrons in the $n = 1$ energy level, so this $n = 2$ electron sees an effective nuclear charge of $Z_{\text{eff}} = Z - 2$. Suppose, however, that an energetic electron knocks out one of the $n = 1$ electrons so that instead $Z_{\text{eff}} = Z - 1$.

[16] *a*) Use the Bohr quantization condition for angular momentum, $L = n\hbar$, to calculate the allowed energy values an electron in a hydrogenic atom of nuclear charge eZ_{eff} can have. (*Hint:* Use Newton's second law, centripetal acceleration, and the Couloumb force law.)

[9] *b*) In such electron-deficient atoms, an electron from the $n = 2$ energy level drops to the $n = 1$ energy level, emitting an x-ray photon. Using the Bohr model from part (*a*), show that the frequency of this emitted photon obeys Moseley's law, $f = M(Z-1)^2$, where M is

$$M = \frac{3m_e e^4}{256\pi^3\epsilon_0^2\hbar^3}.$$

3. Heisenberg uncertainty principle [25 points]

For both parts of this problem, you may assume that the motion of the particle is non-relativistic.

[10] *a*) Starting with the Heisenberg uncertainty principle in position and momentum in one dimension, show that the uncertainty principle for a free particle in one dimension can be written as $\Delta\lambda\Delta x \geq \lambda^2/4\pi$, where $\Delta\lambda$ is the uncertainty in the particle's wavlength and Δx is the uncertainty in the particle's position.

[15] *b*) A particle of mass m moves in a one-dimensional potential

$$V(x) = F_0|x|,$$

where F_0 is a positive constant. Use the Heisenberg uncertainty principle to estimate the minimum total energy (kinetic plus potential) of the particle as a function of m, F_0, and $\hbar$. (*Hint*: Use the principle to express the minimum energy as a function of either momentum or position and take a derivative.)

4. Schrödinger's equation [25 points]

The state of a free particle of mass m in one dimension is described by the following quantum wave function:

$$\psi(x) = \begin{cases} 0 & x < -a \\ A(a - |x|) & -a \le x \le a \\ 0 & x > a. \end{cases}$$

[6] *a*) Determine A using the normalization condition. (You may choose the phase convention so that A is real.)

[6] *b*) What is the probability that a measurement of the particle's position will reveal it to be in the range $[0, a/2]$?

[12] *c*) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.

[1] *d*) Write down Schrödinger's *time-independent* equation for $\psi(x)$. (*Hint*: Remember, it's a *free* particle.)