UNM Physics 262, Problem Set 1, Fall 2006

Instructor: Dr. Landahl
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Due: August 30, 2006

Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. Be sure to put your name or CPS number on your problem set as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

Exercises: Young & Freedman 29.3, 29.17 (Explain why!), 29.24
Problems: Young & Freedman 29.47, 29.49, and the two problems below.
Extra Credit: Young & Freedman Challenge Problem 29.76

I. High frequency capacitors. In Physics 161, using Gauss' Law for electrostatics, you showed that the electric field between two circular parallel plates a distance d apart that carried surface charge densities σ and $-\sigma$ was

$$E = \frac{\sigma}{\varepsilon_0}$$

from the positively charged side to the negatively charged side, neglecting edge effects. (See Example 22.8 in Young & Freedman.) This charge separation can be generated, *e.g.*, by a DC power source connecting the two plates. In this problem, we examine what happens when the plates are connected by an AC power source.

Suppose the plates were driven such that the voltage difference between the top plate and the bottom plate ($V_{\text{top}} - V_{\text{bottom}}$) was

$$V = V_0 \sin \omega t.$$

a) What is the electric field between the plates, $\mathbf{E_1}$, to first approximation? (Remember, $\mathbf{E_1}$ is a vector—specify its direction!)

b) This electric field is changing in time. Using the Maxwell-Ampere Law, develop an expression for the magnetic field, $\mathbf{B_1}$, between the plates. (Hint: Draw pictures! $\mathbf{B_1}$ is a vector!)

c) But wait, this magnetic field is also changing in time! Use Faraday's Law to find the electric field, $\mathbf{E_2}$, generated by this field $\mathbf{B_1}$. (Be sure to draw a picture of your Faraday surface. Hint: along the axis of symmetry of the plates, there is no $\mathbf{E}$ field correction.)

d) Using the principle of superposition, the electric field is now the sum of the expressions $\mathbf{E_1}$ and $\mathbf{E_2}$ found in parts a and c. This is a good approximation for low frequencies, but we want to know what happens when we ramp up the frequencies even higher. The correction $\mathbf{E_2}$ field found in part c is also changing in time. Use the Maxwell-Ampere Law on $\mathbf{E_2}$ to find the next correction, $\mathbf{B_2}$ to the magnetic field.

e) But wait! $\mathbf{B_2}$ is changing in time, which generates a correction to the electric field. Calculate this correction, $\mathbf{E_3}$.

f) Obviously this game goes on ad infinitum. Write down the infinite series for the electric field between the plates. (Work out a few more terms if you need help seeing the pattern.)

g) Use your favorite reference to look up what "Bessel functions" are. (Wikipedia, PlanetMath, Abramowitz & Stegun's text, etc. are examples of where to find this.) Which Bessel function does the series in part f represent? Draw a plot of this function. Bessel functions arise frequently in physics problems involving cylindrical symmetry. They are to cylindrical waves what cosine functions are to waves on a line. Don't be surprised if they show up again in wave optics problems having cylindrical symmetry.

II. The jumping ring demo. In class I presented the "jumping ring" demonstration and told you that the magnetic field in the ring opposed the magnetic field generated by the electromagnet, causing the ring to jump in the air. That explanation isn't the full story because antiparallel magnetic fields cause *torques*, not net forces; see Fig. 27.29 in Young & Freedman and the accompanying discussion for details. A more correct explanation necessarily takes into account the radial part of the field arising because the solenoid is only finitely long. Understanding this explanation will take us on a tour of the principles of electromagnetism.

The current in the coil winding around the solenoid is

$$I = I_0 \sin(\omega t - \theta(z)),$$

where ω is the frequency of the AC current and $\theta(z)$ is the current's phase variation as a function of z, the vertical direction along the solenoid.

a) Use Ampere's Law to calculate the $\mathbf{B}$ field inside the solenoid, approximating the solenoid as being very long relative to its radius. Express your answer in cylindrical coordinates as $\mathbf{B} = B_z\hat{\mathbf{z}} + B_r\hat{\mathbf{r}} + B_\varphi\hat{\varphi}$.

b) Because the solenoid isn't infinitely long, there is a radial component of the magnetic field. Use Gauss' Law for magnetism on a Gaussian cylindrical surface of height dz that is coaxial with the solenoid to relate the radial component of the magnetic field outside the solenoid,

B_r to an appropriate (differential) function of the vertical component of the magnetic field B_z inside the solenoid. Substitute your expression for B_z from part a into this formula and calculate any derivatives.

c) When the (aluminum) ring was placed coaxially with the solenoid in the class demonstration, it acquired an emf from the changing flux of the solenoid's magnetic field. Approximating the solenoid as very long relative to its radius, use Faraday's Law to calculate the emf $\mathcal{E}_r$ generated in the ring.

d) The emf in the ring generates a current via Kirchoff's voltage law. Use this law to express the current I_r in the ring in terms of the emf $\mathcal{E}_r$ and the ring's resistance R. (Hint: In this special case, Kirchoff's law is so simple it also goes by another name.) (Note: This part of the analysis is a bit of a fudge—there is a generalization to Kirchoff's voltage law that we haven't discussed yet that incorporates other voltage drops occurring in the ring, but we will ignore them for this problem.)

e) Calculate the Lorentz force the ring experiences as a result of the current I_r and the radial part of the magnetic field $\mathbf{B} = B_r\hat{\mathbf{r}}$.

f) Integrate the Lorentz force from part e over a cycle to get an expression for the average force experienced by the ring. You should find that it is nonzero and pointing upwards. If you didn't obtain this, go back to the beginning and retrace your steps. Notice how important the phase $\theta(z)$ is to this force. It can be calculated from first principles, but it would mean correcting the fudge made in part d, which we're not ready for yet.

At this point, one could apply Newton's second law to this force and gravity using a free body diagram and follow up with some kinematics to derive the height that the ring will jump. I could have asked you to work through all that in this problem, but I won't. :)