UNM Physics 262, Problem Set 11, Fall 2006

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Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. **Please put your box number on your assignment, which is 952 plus your CPS number**, as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

11.1 Quarkonium pandemonium

In addition to the electron, photon, proton and neutron that we've talked about in class, there are literally hundreds of particles that have been discovered. Most of this "particle zoo" was tamed was by the quark model, put forward by Nobel Laureate Murray Gell-Mann, now a professor here at UNM. Particles that are *fermions* (a concept we will discuss later in this course) can be classified by the number of quarks they contain. They are either leptons (no quarks), mesons (two quarks), or baryons (three quarks).[1] There are six types (called "flavors") of quarks known, labeled u, d, s, c, t, b, and given the respective colorful names *up*, *down*, *strange*, *charmed*, *top*, and *bottom*. For each quark, there is a corresponding antiparticle having the same mass but opposite charge. These antiquarks are symbolized by $\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$, $\bar{t}$, and $\bar{b}$.

Among the mesons, there is a class of particles called *quarkonium*, which are electrically neutral and which consist of a quark and its antiquark bound together. One speaks of "charmonium" ($c\bar{c}$), "bottomonium" ($b\bar{b}$), etc. The situation is very similar to positronium discussed in class with one important difference. The effective central potential the quarks interact by is

$$V(r) = -\frac{4}{3}\frac{\alpha_s \hbar c}{r} + F_0 r. \tag{1}$$

The first term in the potential involves the strong force coupling constant α_s, which is about 0.2 for charmonium The second term accounts for the phenomenon of *quark confinement*: there is an energetic penalty for the quarks getting farther and farther apart, so they stay close together to reduce their energy. The term F_0 is experimentally found to be about 16 tons, so the quarks *really* like to stay together! (F_0 is 900 MeV/fm in more conventional units.)

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[1] There is some theoretical and experimental evidence for pentaquarks, but both theory and experiment on this issue are currently speculative at best.

In this problem, we explore the energy spectrum of charmonium. To make the problem simpler (*i.e.*, to avoid having to solve cubic equations), we neglect the Coulombic term in the charmonium potential, considering instead the simpler central potential

$$V(r) = F_0 r. \tag{2}$$

(*a*) The mass of the charm quark is $m_c = 1.25$ GeV/c^2. What is the reduced mass μ of charmonium in terms of the quark mass m_c? What is its numerical value in GeV/c^2?

(*b*) Use the Bohr quantization condition for the angular momentum, $L = n\hbar$, to obtain an expression for (*i*) the radius and (*ii*) the energy of the nth Bohr orbit in charmonium's equivalent one-body problem. (Hint: The central force is given by $F = -dV/dr$.)

(*c*) Suppose a charmonium meson drops from energy level n to energy level $n-1$, emitting a photon in the process. (*i*) What is the frequency ν of the radiation? Express your answer in terms of F_0, μ, $\hbar$, and n. (*ii*) What is the "Lyman series" associated with emission from energy level n to energy level 1? What is the associated "Rydberg constant" R? Evaluate both R and R^{-1} numerically in SI units.

(*d*) In classical electromagnetism, a charged object undergoing oscillatory motion of period T and frequency $\nu = 1/T$ emits light at frequency ν. For charmonium in the Bohr orbit labeled by n, what is the classical prediction for the frequency of the radiation the (charged) charm quarks emit?

(*e*) Numerically evaluate the frequency of the emitted light for a charmonium transition from the energy level $n = 2$ to the energy level $n = 1$ for (*i*) the quantum model derived in part (*c*), and (*ii*) the classical model derived in part (*d*). (Hint: To avoid recalculating numbers, you may want to use your expression for the "Rydberg constant" of charmonium.)

(*f*) In the $n \to \infty$ limit, what is the ratio of the two frequencies above? (Hint: Use the binomial approximation formula.) The *Bohr correspondence principle* asserts that for large n, quantum and classical calculations should agree. Are your quantum and classical calculations for the frequency of emitted radiation from charmonium consistent with this principle?

(*g*) The energy levels of charmonium are substantial in relation to the rest masses of the charm quarks themselves. For this reason, different particles are associated with different excited states of charmonium, with the total rest mass of the particle being equal to the sum of the charm quark masses plus the interaction energy. For example, the first charmonium meson discovered was the J/ψ particle, which corresponds to the $n = 2$ excited state of charmonium.[2] Using your formula for the energy of charmonium from part (*b*), what do you find the total mass of the J/ψ particle to be, in GeV/c^2? What fractional error does this estimate have with the experimentally determined value of 3.097 GeV/c^2 for the total mass of charmonium? (f = (theory − experiment)/experiment.) What fractional error is there with experiment if the theory neglects the rest mass associated with the excited level entirely?

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[2] This strange name for the particle reflects a historical priority dispute regarding the discovery of the particle: one camp (MIT) called it J while the other (Stanford) called it ψ.

11.2 Rutherford scattering

When Geiger and Marsden scattered α particles, of charge $2e$, off of gold atoms, they witnessed large scattering angles that could not be accounted for by Thomson's "plum pudding" model of the atom. Rutherford explained these results by instead modeling each gold atom as having its positive charge Ze lie entirely at a single point at its center. (See extra credit problem 11.6 to see more exactly how the scattering angles predicted by this model depend on the parameters above.)

For incident α particles with sufficiently high energy, Rutherford's model no longer accurately predicts the data of Geiger and Marsden's experiment. Rutherford realized that this indicated that the positive charge of the atom doesn't lie at an infinitesimal point but rather in a *nucleus* of finite size at the atom's center. Such high-energy α particles can actually penetrate the nucleus and will in general experience a different potential than the one associated with the Coulomb force. (See problem 11.1 for an example of such a new potential.) Turning the failure of his model into a success, he used Geiger and Marsden's data to estimate the size of the nucleus of the gold atom. We reproduce his calculation here.

(a) The lowest-energy α particles that failed to obey Rutherford's scattering formula had an initial kinetic energy of 32 MeV. What is the distance of closest approach (in SI units) of such particles incident upon gold atoms ($Z = 79$)? Neglect the recoil of the gold nucleus. (Hint: Use conservation of energy.) Give a qualitative argument why the electrons surrounding the gold nucleus should or should not be ignored in this calculation.

(b) The so-called Fermi gas model of the nucleus predicts that the radius of an atom's nucleus should be $r = r_0 A^{1/3}$, where $r_0 = 1.2$ fm and A is the atomic mass number of the atom. For the most abundant isotope of gold, ^{197}Au, what does the Fermi gas model predict that gold's nuclear radius should be?

(c) The experimentally determined atomic radius of the gold atom is about 135 pm. What is the ratio of this to the radius of the gold nucleus you deduced in (a)? Compare this to the ratio of the Earth's orbital radius around the Sun to the radius of the Sun.

11.3 Work functions

A material sample is heated to a temperature T in vacuum and a thermionic emission current density of J is measured to emanate from the sample. The sample is then removed and placed in an photoelectric effect experiment where it is irradiated with light of a tunable frequency. Write down an expression for the minimum frequency of the light that will cause photoelectron emission from the sample. Your answer should be expressed as a function of $\hbar$, e, m_e, k_B, J, and T. (This problem relies on *Richardson's law*, which was discussed in class but is not in the book. Didn't take good notes? Look it up on the Internet!)

11.4 Radiative collapse of a classical atom

Rutherford's classical "solar system" model of the atom is unstable. The problem is that an accelerating charge, such as the electron orbiting the nucleus in his model, radiates energy at a rate given by the following *Larmor formula*:

$$\frac{dE}{dt} = -\frac{2}{3}\frac{q^2}{4\pi\epsilon_0}\frac{a^2}{c^3},$$

where q is the electric charge, a is the magnitude of the acceleration, and c is the speed of light.

In this problem we consider Rutherford's model as applied to hydrogen.

(a) Show that the energy lost per revolution by the electron is small compared to the electron's kinetic energy. Hence, it is an excellent approximation to regard the orbit as circular at any instant, even though the electron eventually spirals into the proton.

(b) Using the typical size of an atom (0.1 nm) and a nucleus (1 fm), arrive at a crude estimate of how long it would take for the electron to spiral into the proton by assuming the electron loses the same kinetic energy per revolution during its entire inspiral until none of the original kinetic energy of the electron remains. (This rough estimate also assumes that the period of each revolution is unchanged during inspiral.)

(c) A better estimate of this "radiative collapse time" accounts for the fact that during inspiral, the orbital period is changing, albeit slowly. Obtain a more refined estimate of this time by taking this effect into account. (Hint: First express the total energy of the hydrogen atom as a function of the electron's orbital radius r. Then take a derivative with respect to time and equate it to the Larmor formula. Integrate to obtain Δt in terms of the initial and final radii of the collapse.)

(d) Using the values for the typical sizes of atoms and nuclei from part (b), compute the velocity v_i/c of the electron initially, as a fraction of the speed of light. Compute the final velocity v_f/c of the electron when it strikes the nucleus. Is relativity needed to get reliable answers for this problem?

11.5 Bremsstrahlung

(a) If a photon has an energy equal to at least twice the rest mass energy of the electron, a process called *pair creation* can begin to occur. In this process, a photon disintegrates and creates an electron-positron pair. The reason that this particular pair of particles is created is threefold: i) by conservation of momentum, more than one massive particle needs to be created, ii) by conservation of charge, the net charge of the particles created must be zero and iii), the electron (and positron) are the lightest of all massive particles. What is the minimum voltage needed across an X-ray tube if the subsequent bremsstrahlung radiation is to be capable of pair production?

(b) A 20 keV electron emits *two* bremsstrahlung photons as it is being brought to rest in two successive decelerations. The wavelength of the second photon is 0.13 nm longer than the wavelength of the first. (i) What was the energy of the electron after the first deceleration, and (ii) what are the wavelengths of the photons?

Extra Credit Problems

11.6 More on Rutherford scattering

For two objects approaching one another not on along the line connecting them but slightly off-center, the impact parameter b of the approach is the perpendicular distance between the two objects. (If there were no interactions between the objects, the distance of closest approach would be b.)

Show that Rutherford's model of the atom predicts that the scattering angle θ of particles of mass m, charge ze, and impact parameter b incident on atoms of nuclear charge Ze is

$$\tan\frac{\theta}{2} = \frac{g}{bmv^2},$$

where g is the constant

$$g = \frac{zZe^2}{4\pi\epsilon_0}.$$

(Hint: Exploit symmetry to choose a good orientation of axes to solve the problem in, and use the conservation of angular momentum. No quantum physics is needed to solve this problem.)

11.7 More on quarkonium

Use the more correct quarkonium potential of Eq. (1) to work out what the radius of the first Bohr orbit ($n = 1$) is for charmonium (the J/ψ particle).