

## UNM Physics 262, Problem Set 12, Fall 2006

Instructor: Dr. Landahl

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Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. **Please put your box number on your assignment, which is 952 plus your CPS number**, as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

### 12.1 Let's hear it for de Broglie

(a) Show that the de Broglie wavelength of a particle of charge  $e$  and rest mass  $m$  moving at a relativistic speed is given as a function of the accelerating potential  $V$  as

$$\lambda = \frac{h}{\sqrt{2meV}} \left( 1 + \frac{eV}{2mc^2} \right)^{-1/2}. \quad (1)$$

(Hint:  $p = \gamma mv$  in special relativity.)

(b) Show that Eq. (1) agrees with  $\lambda = h/p$  in the nonrelativistic limit. (Hint: Use the binomial expansion.)

(c) At what energy, in electron volts, will the nonrelativistic expression for the de Broglie wavelength be in error by 1% for (i) an electron? (ii) a neutron?

### 12.2 Building better microscopes

The resolving power of a microscope is limited only by the wavelength used; that is, the smallest detail that can be seen is about equal to the wavelength. Consider the following objects one might investigate with a microscope:

- A virus of diameter 10 nm.
- An atom of diameter 0.1 nm.
- A proton of diameter 1.0 fm.

(a) If an electron microscope is used, to what voltages must the electrons be accelerated to resolve each of the three objects? Answer for each object separately.

(b) If a photon microscope is used, what photon energy, in electron volts, is needed to resolve each of the three objects? Answer for each object separately.

(c) Which microscope seems most practical for each of the objects and why?

### 12.3 Heisenberg uncertainty for nuclear physics

(a) The  $J/\psi$  particle described in problem set 11 is unstable. (It is the  $n = 2$  excitation, recall). It has a rest mass of  $3.097 \text{ GeV}/c^2$  and a (proper) lifetime of  $7.6 \times 10^{-21} \text{ s}$ . What is the uncertainty in the rest mass of the  $J/\psi$  particle as a percentage of its rest mass?

(b) The *neutron* is an electrically neutral particle with a mass approximately equal to the proton mass. An early model of the neutron was that of electron confined inside the proton. Using a proton radius of  $r = 10 \text{ fm}$ , (i) estimate the electron's kinetic energy due to Heisenberg uncertainty and (ii) compare it to the neutron rest mass. What is your assessment of this model? (Think: do you need to use relativity for this problem?)

(c) A proton or neutron can sometimes “violate” energy conservation by emitting and then reabsorbing a pi meson, or *pion*, which has a mass of  $m_\pi = 135 \text{ MeV}/c^2$ . This is possible as long as the pion is reabsorbed within a short enough time  $\Delta t$  consistent with the Heisenberg uncertainty principle. (i) For how long can such a “virtual” pion exist? (Ignore kinetic energies.) (ii) Assuming the pion travels at very nearly the speed of light, how far from the proton or neutron can it go? (This gives an estimate of the range of the nuclear force, because we believe protons and neutrons are held together in the nucleus by the exchange of pions.)

### 12.4 Zero point energy of the hydrogen atom

For some quantum systems, the lowest energy configuration predicted by quantum mechanics differs from the one predicted by classical mechanics because of the Heisenberg uncertainty principle. For example, one way to theoretically estimate the size of the hydrogen atom is to calculate how large a radius the atom can have in its lowest quantum mechanical energy configuration. In this problem, we go through this calculation.

(a) Write down the classical expression for the total energy of the hydrogen atom with an electron of momentum  $p$  in a circular orbit of radius  $r$ . Keep the kinetic and potential energy terms separate at this point.

(b) Now use Newton's second law and the Coulomb force to rewrite the classical energy of the hydrogen atom entirely in terms of its radius. What radius corresponds to the lowest possible energy?

(c) In the lowest energy quantum mechanical configuration of the hydrogen atom, the momentum of the electron (which is entirely azimuthal) and its location along the circumference of its orbit (the conjugate variable to this momentum) have approximately the same magnitudes as their respective uncertainties, the product of which is lower bounded by the Heisenberg uncertainty principle. For this lowest-energy state, express the total energy of the hydrogen atom entirely in terms of its radius.

(d) Take the derivative of this energy with respect to the radius and set it to zero to

find the radius that minimizes the total energy. How does this quantity compare to the expression for the Bohr radius of the hydrogen atom?

## 12.5 Gaussian wave functions

A particle of mass  $m$  has the quantum wave function

$$\Psi(x, t) = Ae^{-a[(mx^2/\hbar)+it]}, \quad (2)$$

where  $A$  and  $a$  are positive real constants. Such a wave function is called a *Gaussian* wave function because the spatial part of the wave function looks like a “Bell curve,” also known as a *Gaussian function* in mathematics.

(a) Because  $P(x, t) = |\Psi(x, t)|^2$  is a probability distribution, its integral over all space must equal one for all times. Use this fact to determine the constant  $A$ .\*

(b) For what potential energy  $V(x)$  does  $\Psi$  satisfy the Schrödinger equation?

(c) Again, because  $P(x, t)$  is a probability distribution, the expected value of a function  $f(x)$  is given by

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)P(x)dx.$$

Compute (i)  $\langle x \rangle$ , (ii)  $\langle x^2 \rangle$ , and (iii)  $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$  using this wave function.\*

\* For parts (a) and (c), you may find the following integral and identity helpful:

$$I = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \qquad \int_{-\infty}^{\infty} (-x^2)^n e^{-\alpha x^2} dx = \frac{d^n I}{d\alpha^n}.$$

(Hint: The integral of an odd function over all space is zero.)