UNM Physics 262, Problem Set 3, Fall 2006

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Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. Be sure to put your name or CPS number on your problem set as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

Shorter "Exercises"

3.1. Objects in mirror are closer than they appear. In the movie *Jurassic Park*, a car is chased by a T. Rex, and the camera pans to the image of the T. Rex in the car's convex side view mirror, displaying the humorous caption "Objects in mirror are closer than they appear." Suppose the T. Rex is $p = 100$ m behind the mirror and is running at a speed $v_p = 10$ m/s towards it. Let the radius of curvature of the mirror be $r = 2$ m. (a) With what velocity v_i does the image of the T. Rex appear to be running when viewed in the mirror? (b) Show that the general expression for the velocity of the T. Rex image is

$$\vec{\mathbf{v}}_i = -\left(\frac{r}{2p - r}\right)^2 \vec{\mathbf{v}}_p. \tag{1}$$

(*Hint*: Start with the image equation for spherical mirrors, Eq. 34.4 in the book.) (c) How far is the T. Rex from the mirror when $\vec{\mathbf{v}}_i = -\frac{1}{2}\vec{\mathbf{v}}_p$? ($d$) How far is the T. Rex from the mirror when $\vec{\mathbf{v}}_i = -\frac{9}{10}\vec{\mathbf{v}}_p$?

3.2. National Treasure. In the National Archives in Washington, D.C., the Declaration of Independence can be viewed by tourists from behind a pane of bulletproof glass ($n_g = 1.50$). The thickness of the glass is 2 cm and there is an air gap between the glass and the Declaration of 3 cm. How far beneath the top of the glass plate does the writing appear to be when viewed head-on? Is this closer or farther away from this surface than the Declaration actually is?

3.3. Hey six eyes! Dilbert the lovable geek is finding that his eyesight is becoming quite poor. The focal length of his current pair of eyeglassess is f_1 and the focal length of his old pair of eyeglasses is f_2. Rather than purchase a new pair of eyeglasses, self-reliant Dilbert decides to take his old lenses out of their frame and attach them directly to his current glasses so that along the lenses' common optic axis, they are osculating. (If you don't know what osculating means, look it up!) (*a*) Show that the focal length of Dilbert's new pair of eyeglasses is

$$f = \frac{f_1 f_2}{f_1 + f_2}. \tag{2}$$

The *power* of a thin lens is measured in diopters, which has units of m^{-1} and is defined as $P = 1/f$, where f is the focal length of the lens. (*b*) Show that the formula from part (*a*) means that Dilbert's new homemade eyglasses have power $P = P_1 + P_2$, where $P_1 = 1/f_1$ and $P_2 = 1/f_2$.

3.4. Optics in daily life. (*a*) A dentist wants to buy a small mirror that will produce an upright image of lateral magnification $m = 5$ when placed $p = 3$ cm from a tooth. (*i*) What radius of curvature r of mirror should she buy? (*ii*) Should she request a concave or convex mirror? (*b*) A shaver wants a bathroom mirror that will produce an upright image of lateral magnification $m = 1.5$ when the mirror is $p = 30$ cm away from his face. (*i*) What focal length f should this mirror have? (*ii*) Should the mirror be concave or convex? (*c*) A photographer has a camera with a converging thin lens of focal length 50 mm. To create an image size of 24 mm on the film, how far should a 1.75 m person stand from the camera?

Longer "Problems"

3.5. Scuba adventure. You are about to go on a scuba dive and you are wondering how well your eyeglasses will work underwater if you (unwisely) abandon your facemask. (*a*) Show that if f is the focal length of a thin lens in air, then its focal length in water is f' given by

$$f' = \frac{n_w(n-1)}{n - n_w} f. \tag{3}$$

(*Hint*: Rework the lensmaker's equation in water.) (*b*) Calculate the focal length in air and in water of a double-concave lens of index of refraction $n = 1.5$ that has radii of magnitude 30 cm and 32 cm.

3.6. Somewhere over the rainbow. A ray of yellow Sodium light ($\lambda = 589$ nm) enters normally through a non-hypotenuse edge of a right angle prism having opposite angle α, as depicted in Fig. 33.36 in the textbook. The prism is made of fused quartz. (*a*) If the prism is immersed in air, what is the largest value of the angle α such that the light is totally reflected? (*b*) If it is immersed in water? (*Hint*: Use Table 33.1 in the book to find the indices of refraction.) (*c*) The ray of yellow light is replaced by a ray of white light. If all the color components of the light undergo total internal reflection at the surface, then the reflected light forms a reflected ray of white light. However, if the color component at one end of the visible range (either red or blue) partially refracts through the surface into the air, there is less of that component in the reflected light. Then the reflected light is not white but has the tint of the opposite end of the visible range. (If blue were partially lost to refraction, then the reflected beam would be reddish, and vice versa.) Is it possible for the reflected light to be reddish? (Why?) (*d*) Is it possible for the reflected light to be bluish? (Why?) (*Hint*: Consider Fig. 33.16 in the book.) (*e*) For unpolarized light incident from the air onto the hypotenuse of the prism, at what incidence angle θ_i will the reflected light be totally polarized, as a function of the prism angle α that is critical for total internal reflection found in part (*a*)?

Extra Credit Problems

3.7. My, what a big nose you have! If you look into a convex mirror, you'll notice that not only have the lateral sizes of your features changed, but also the longitudinal sizes of your features have changed. Fig. 33.16 in the book shows an example where Santa's nose looks not only wider than his feet, but looks foreshortened as well. Recall that for thin lenses and mirrors, the magnification of an object a distance p from the surface having an image a distance i from the surface is $m = -i/p$, subject to the sign conventions for i and p we are using. (*a*) Show that the *longitudinal magnification* m_L of an object of small longitudinal extent through a thin lens or spherical mirror is approximately $-m^2$. (*Hint*: Show that $di/dp = i^2/p^2$.) (*b*) What is the significance of the minus sign in this formula? (*c*) For a spherical *refracting* surface, show that the longitudinal magnification is

$$m_L = -\frac{n_t}{n_i} m^2, \tag{4}$$

where light propagates from an object in a medium of index of refraction n_i to an image in a medium of index of refraction n_t.

3.8. Finding Nemo. In a miraculous feat of animated computer graphics, you've managed to capture Nemo the fish and put him in your lovely fish bowl. Approximate the fish bowl as a perfect sphere filled completely with water ($n_w = 1.33$) and having glass walls ($n_g = 1.50$). The inner radius of the sphere is $r_1 = 10$ cm and the outer radius is $r_2 = 11$ cm. (In other words, the glass is 1 cm thick.) (*a*) When Nemo approaches you at distance $a = 5$ cm from the center of the bowl, where do you see his image? (*b*) When Nemo backs away from you and is a distance $a = -5$ cm from the center of the bowl, where do you see his image? (*Hint*: For parts (*a*) and (*b*), Nemo's image refracts through two interfaces before it reaches you. The image of the first refraction is the object for the second refraction. Use the sign conventions for object, image, and radius distances to keep things straight.) (*c*) The fishbowl acts as a "thick lens" for light striking it from outside the bowl and has a corresponding focal length. Find the focal length of the bowl. (*Hint*: Consider an object at infinity that passes through four hemispherical interfaces: air-glass, glass-water, water-glass, glass-air.) (*d*) Is it possible to replace the glass with a material with a different index of refraction n_{wall} so that the focus is *inside* the fish bowl? If so, what is the smallest n_{wall} that will permit this? Why might Nemo not be so happy about the focal length being inside the fishbowl?