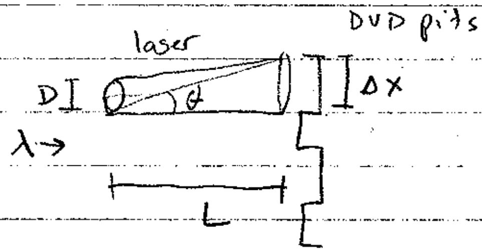
Ph267 PS5

(5.1)

(a)

SPY DVD




Diffraction limited beam: $\sin\theta \sim \theta \sim \frac{\Delta x}{L} = \lambda/D$

$\Delta x_{DVD} = \frac{L\lambda_{DVD}}{D}$

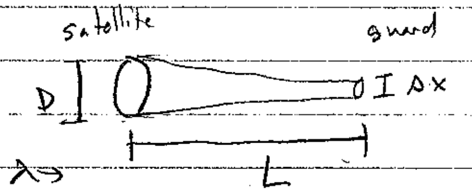
$\Delta x_{spy\ DVD} = \frac{1}{2}\Delta x_{DVD}$ (spy DVD has pits half as wide)

$\Rightarrow \lambda_{spy\ DVD} = \frac{D\Delta x_{spy\ DVD}}{L} = \frac{1}{2}\left(\frac{D\Delta x_{DVD}}{L}\right) = \frac{1}{2}\lambda_{DVD}$

$\lambda_{spy\ DVD} = \frac{1}{2}(640\text{ nm}) = \boxed{320\text{ nm}}$

(b)

Spy satellite




$D = 2.5$ m
$\lambda = 550$ nm
$\Delta x = 0.5$ m

Diffraction-limited resolving: $\lambda/D = \theta \sim \Delta x/L$

$L = \frac{D\Delta x}{\lambda} = \frac{(2.5\text{ m})(0.5\text{ m})}{(550\text{ nm})} = \left(\frac{1.25}{5.5\times10^{-7}}\right)\text{ m}$

$L = 2.3\times10^{6}\text{ m} = \boxed{2300\text{ km}}$

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(cont.)

5.1

(c) The angle in part (c) is

$$\theta = \frac{\Delta x}{L} = \frac{0.5\,m}{2.3\times10^{6}\,m} = 2.2\times10^{-7}\ \underline{\text{radians}}$$

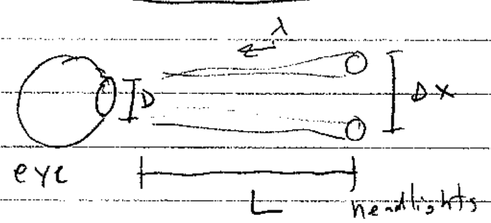
Converting to arcseconds, this is

$$2.2\times10^{-7}\,\text{rad}\ \Big|\ \frac{360^\circ}{2\pi\ \text{rad}}\ \Big|\ \frac{60\ \text{arcmin}}{1^\circ}\ \Big|\ \frac{60\ \text{arcsec}}{1\ \text{arcmin}}$$

$$\boxed{\theta = 4.5\times10^{-2}\ \text{arcsec}} \quad < 0.3\ \text{arcsec}$$

Adaptive optics must be used at this height.

(d)




Motorcycle or car?

$\lambda = 550\,nm$

$D = 5\,mm$

$\Delta x = 1.5\,m$

For the headlights to be resolved, the diffraction limit yields:

$$\lambda/D = \theta \sim \frac{\Delta x}{L}$$

$$L = \frac{D\,\Delta x}{\lambda} = \frac{(5\,mm)(1.5\,m)}{(550\,nm)} = \frac{(5\times10^{-3})(1.5)}{5.5\times10^{-7}}\,m = \frac{7.5}{5.5}\times10^{-3+7}\,m$$

$$= 1.4\times10^{4}\,m = \boxed{14\ km}$$

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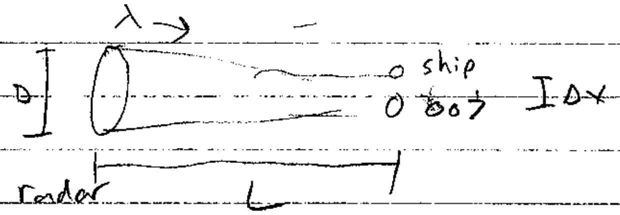
(still cont.)

5.1

Radar evasion

(e)




$\lambda = 3.2$ cm

$D = 2.3$ m

$L = 1$ mi $= 1.6$ km

For Bond to be hidden in the radar's diffraction limit,

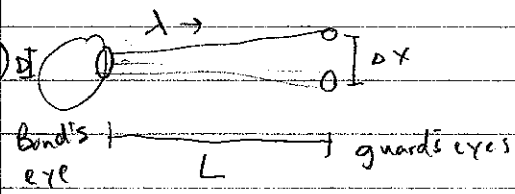
$$\lambda/D \sim \theta \sim \frac{\Delta x}{L}$$

$$\Delta x = \frac{L\lambda}{D} = \frac{(1.6\text{ km})(3.2\text{ cm})}{2.3\text{ m}} = \frac{(1.6\times10^3\text{ m})(3.2\times10^{-2}\text{ m})}{2.3\text{ m}}$$

$$= \frac{(1.6)(3.2)}{2.3}\times10^{1}\text{ m}$$

$$\boxed{\Delta x = 22\text{ m}}$$

(f)




Guard's eyes

$\lambda = 800$ nm

$D = 5$ mm

$\Delta x = 6.5$ cm

For the guard's eye separation to be resolvable (which admittedly isn't really a "whites of their eyes" criterion), the diffraction-limited resolvability is

$$\lambda/D \sim \theta \sim \frac{\Delta x}{L}$$

$$L = \frac{D\Delta x}{\lambda} = \frac{(5\text{ mm})(6.5\text{ cm})}{800\text{ nm}} = \frac{(5\times10^{-3})(6.5\times10^{-2})}{8\times10^{-7}}\text{ m}$$

$$= \frac{5(6.5)}{8}\times10^{-5+7} = \frac{5(6.5)}{8}\times10^{2}\text{ m} = 4.1\times10^{2}\text{ m}$$

$$\boxed{L = 410\text{ m}}$$

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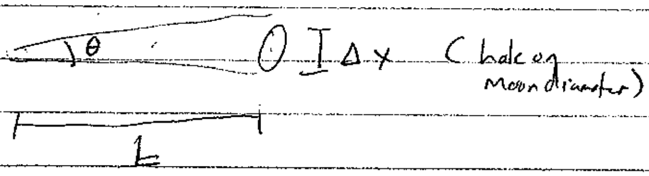
Hole in moon

5.1 (9)




$L = 3.84 \times 10^8$ m

$\theta = 0.3$ arcsec

For the bore size not to be seen at $\theta = 0.3$ arcsec, we need

$$\theta \gtrsim \Delta x / L \Rightarrow \Delta x \lesssim L\theta$$

in radians

$$\Delta x \lesssim (3.84 \times 10^8 \text{ m}) \left( \frac{0.3 \text{ arcsec}}{} \cdot \frac{1 \text{ arcmin}}{60 \text{ arcsec}} \cdot \frac{1^\circ}{60 \text{ arcmin}} \cdot \frac{2\pi \text{ rad}}{360^\circ} \right)$$

$$\Delta x \lesssim 5.6 \times 10^2 \text{ m}$$

$$\boxed{\Delta x \lesssim 560 \text{ m}}$$

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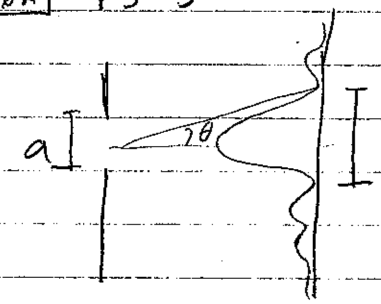
5.2 (a)




$V = 330$ m/s
$f = 1250$ Hz

The far-field intensity pattern is

$$I = I_0 \operatorname{sinc}^2 \phi \quad , \text{ where } \phi = \frac{\pi a \sin\theta}{\lambda}.$$

The wavelength of sound is from $v = f\lambda$:

$$\lambda = V/f = \frac{330 \text{ m/s}}{1250 \text{ s}^{-1}} = 2.64 \times 10^{-1} \text{ m} = 26.4 \text{ cm}$$

Intensity minima occur when $\operatorname{sinc}^2 \phi = 0$, which is at

$$\sin \phi = 0 \iff \phi = 0 + \pi m$$

$$\frac{\cancel{\pi} a \sin\theta}{\lambda} = \cancel{\pi} m$$

$$\sin\theta = \frac{m\lambda}{a}$$

To have the first minimum (m=1) never occur, we need

$$\frac{\lambda}{a} \geq 1 \Rightarrow \lambda \geq a.$$

Hence we need $\boxed{a \leq 26.4 \text{cm}}$ for only 1 maxima to exist.

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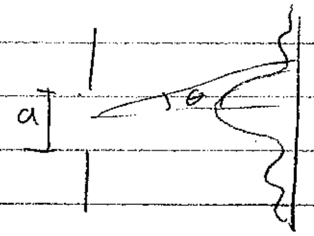
(5.2) (b)



$a = 1.0\ m$

$\lambda = 26.4\ cm$ (from part (a))

As shown in part (a), intensity minima occur for

$$\sin\theta = \frac{m\lambda}{a}$$

Since $\frac{\lambda}{a} = \left(\frac{26.4\ cm}{1.0\ m}\middle|\frac{m}{100\ cm}\right) = 0.264$,

we have

$$\boxed{\begin{aligned} \theta_1 &= \sin^{-1}(0.264) = 0.267\text{ rad} \quad (15.3^\circ) \\ \theta_2 &= \sin^{-1}(0.528) = 0.556\text{ rad} \quad (31.9^\circ) \\ \theta_3 &= \sin^{-1}(0.792) = 0.914\text{ rad} \quad (52.4^\circ) \end{aligned}}$$

$\theta_4 = \sin^{-1}(1.056) \rightarrow$ doesn't exist

(c) The speaker is taller than it is wide so it will diffract more horizontally than vertically.

(d) The width is the same as in part (a), because it is the same problem:

$$\boxed{26.4\ cm\text{ wide}}$$

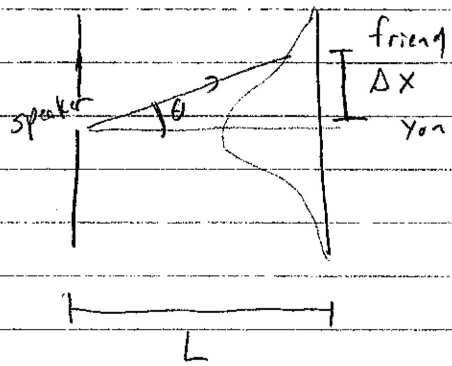
Ph262 PS 5

(5.2) (e) Using the single-slit diffraction formula

$$I = I_0 \ \mathrm{sinc}^2 \phi ,$$

$$\phi = \frac{\pi a \sin\theta}{\lambda} ,$$

And the following geometry to find $\sin\theta$




$\Delta X = 5.0\,m$

$L = 10.0\,m$

$\tan\theta = \frac{\Delta X}{L} = \frac{1}{2}$

$\sin\theta = \frac{\Delta X}{\sqrt{L^2 + (\Delta X)^2}}$

we can calculate the intensity at the friend's location, using the following numbers:

$I_0 = 1.0\,W/m^2$

$a = 26.4\,cm$

$\lambda = 26.4\,cm$

$$I_{friend} = I_0 \frac{\sin^2\left(\frac{\pi a}{\lambda} \frac{\Delta X}{\sqrt{L^2+\Delta X^2}}\right)}{\left[\frac{\pi a}{\lambda} \sin^{-1}\left(\frac{\Delta X}{\sqrt{L^2+(\Delta X)^2}}\right)\right]^2}$$

In radians

$$= \frac{(1.0\,W/m^2)\ \sin^2\left(\frac{\pi\ (5.0\,m)}{\sqrt{(10.0\,m)^2+(5.0\,m)^2}}\right)}{\pi^2\left[\sin^{-1}\left(\frac{(5.0\,m)}{\sqrt{(10.0\,m)^2+(5.0\,m)^2}}\right)\right]^2} = \frac{9.728\times10^{-1}\,W}{2.122}$$

$$\boxed{I_{friend} = 0.46\,W/m^2}$$

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5.2 (f) Since the width of a mouth is less than $\lambda = 26.4\,cm$, there are no intensity minima.