UNM Physics 262, Problem Set 6, Fall 2006

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Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. **Please put your box number on your assignment, which is 952 plus your CPS number**, as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

6.1 IRFs in NIRFs. In class, I discussed a hypothetical situation in which a RailRunner train car is launched straight up into the air from the New Mexico Spaceport. Strictly speaking, an infinite lattice of synchronized clocks and meter sticks comoving with this train car defines a non-inertial reference frame (NIRF). This is because tidal gravitational forces generated by the Earth will cause objects ("test particles") initially at rest in this frame to exhibit accelerations, thereby violating Newton's First Law. Nevertheless, I argued that this frame could serve as a decent inertial reference frame (IRF) over time and space scales so short that such accelerations were undetectable to within some specified experimental precision. In this problem, we investigate this argument more critically.

(a) Consider two steel spheres near the surface of the Earth and originally separated horizontally by 20 m. (They can be freely falling in the air; they don't need to be in the RailRunner car.) Demonstrate that the spheres move closer together by approximately 1 mm as they fall 315 m, using the following similar triangles method (or some other method).

Similar triangles method: Draw a picture depicting points for the center of the first sphere (A), the center of the second sphere (B), and the center of the Earth (C). Draw the line segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Label a point (D) halfway along the line segment connecting A and B. Also draw an arc indicating where the surface of the Earth is. Now draw a line parallel to $\overline{AB}$ that is tangent to the surface of the Earth. Drop a line perpendicular to $\overline{AB}$ from the point B and label the point where it intersects the line tangent to the Earth's surface (E). Finally, label the point where the line $\overline{BC}$ intersects the Earth's surface (F).

From this picture, one can see that the length of the line segment $\overline{EF}$ is equal to *half* the change in horizontal separation the spheres experience as they fall. One can also see that BDC and BEF are similar triangles. Using this similarity property, and using the problem's inputs ($\overline{AB} = 20$ m, $\overline{BE} = 315$ m), calculate this half-change and double it to show that the full separation the spheres experience is approximately 1 mm. (N.B., The radius of the Earth is in the front flap of Ohanian's textbook.)

(b) Now consider two steel spheres near the surface of the Earth that are originally separated *vertically* by 20 m. If the sphere closest to the Earth is initially 315 m above the Earth, demonstrate that the two spheres increase their separation by approximately 2 mm before the first sphere hits the Earth. You may find the following outline useful.

Problem-solving outline: Take the gravitational acceleration at the surface of the Earth to be $g_0 = 9.8\ \mathrm{m/s^2}$. The gravitational acceleration of an object of mass m a distance r from the center of the Earth (mass M, radius R) is given by the expression

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m} = -\frac{GM}{r^2}\hat{\mathbf{r}} = -\frac{GM}{R^2}\frac{R^2}{r^2}\hat{\mathbf{r}} = -\frac{g_0 R^2}{r^2}\hat{\mathbf{r}}.$$

Using $\vec{\mathbf{g}} = -g\hat{\mathbf{r}}$, this equation becomes

$$g = \frac{g_0 R^2}{r^2}.$$

Take the differential of this equation for g to get an expression dg for how a small change in g depends on a small change in height dr. Then take the differential of the equation $y = \frac{1}{2}gt^2$ to get an expression dy for how a small change in the vertical separation y between the spheres depends on a small change in the local gravitational acceleration dg. Combine these equations to express dy in terms of dr. Finally use the equation $h = \frac{1}{2}gt^2$ and solve for t to express the time it takes for the bottom sphere to hit the Earth in terms of of its original height h. (The actual time is slightly different because gravity is not uniform vertically, but the difference is unimportant for this analysis.) Substitute in the numbers from the problem and you should find the 2 mm separation quoted.

(c) As a concluding part of this analysis, consider two steel spheres that are separated by 20 m, far from any gravitational influences such as that of the Earth. Because they are massive, they will generate a gravitational acceleration towards one another that will appear as a violation of Newton's First Law. (Their utility as "test particles" is limited.) Using the time found in part (b) for a sphere to hit the Earth when dropped from 315 m, what is the largest radius that each of these two identical steel spheres can have such that their separation changes by less than 1 mm in this amount of time due to their mutual gravitational attraction? The density of steel is approximately $8000\ \mathrm{kg/m^3}$.

6.2 The Principle of Relativity. (a) Two overlapping IRFs are in uniform relative motion. According to the Principle of Relativity, which of the following quantities must *necessarily* be the same as measured in the two frames? Explain your answers for each of the five cases.

1. The speed of an electron.
2. The charge of an electron.
3. The mass of an electron.
4. The kinetic energy of an electron.
5. The electric field of an electron.

(b) The work-energy theorem states that the change in kinetic energy of an object is equal to the work done on it, which is measured by the dot product of the force on the object with the vector distance through which it was displaced. In coordinates, Alice expresses this law of physics in her IRF as:

$$\Delta KE - \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{x}} = 0.$$

Bob is in an IRF moving with a velocity $\vec{\mathbf{v}}_{rel}$ in Alice's positive x-direction. In Bob's IRF, he writes down the corresponding expression for the left hand side of the work-energy theorem in his (primed) coordinates as:

$$\Delta KE' - \vec{\mathbf{F}}' \cdot \Delta\vec{\mathbf{x}}'.$$

Use the Galilean transformation to rewrite this expression totally in terms of Alice's coordinate variables. (Careful: the theorem uses *vectors*.) Show that the expression necessarily vanishes, thereby upholding the Principle of Relativity for this law of physics.

6.3 Relativity of wind. (a) Southwest Airlines (SWA) has a fleet of identical airplanes that all have the same air speed c (which is *not* the speed of light!). One day, as you are flying from Albuquerque (A) to Boston (B) on SWA, there is a stiff wind of speed v blowing from B toward A for the entire flight. On the return SWA trip, the exact same wind is blowing. (i) Show that the time for the round trip from A to B and back under these circumstances is greater by a factor $1/(1 - v^2/c^2)$ than the corresponding round trip in still air. (ii) There's a seeming paradox here: The wind helps on one leg of the flight as well as hinders on the other. Why, therefore, is the round-trip time not the same in the presence of wind as in still air? Give a simple physical reason for this difference. It may be helpful to consider the following limiting case: What happens when the wind speed is nearly equal to the speed of the airplane?

(b) On a later SWA flight to Calgary (C) from Albuquerque, the same wind is blowing from B to A, but this time the wind is perfectly perpendicular to the path of the airplane. Again, on the return SWA trip, the same wind is blowing. Show that the time for the round trip from A to C and back under these circumstances is greater by a factor $1/\sqrt{1 - v^2/c^2}$ than the corresponding round trip in still air.

(c) Two SWA airplanes leave from A at the same time. One travels from A to B and back to A, flying first against and then with the wind (wind speed v) The other travels from A to C and back to A, flying across the wind. Assume the distances between A and B and A and C are the same. (i) Which one will arrive home first and what will be the difference between their arrival times? (ii) Using the first two terms of the binomial theorem (or equivalently the first two terms of the Taylor expansion),

$$(1 + x)^n \approx 1 + nx \quad \text{for } |x| \ll 1,$$

show that if $v \ll c$, then the approximate expression for this time difference is $\Delta t \approx (L/2c)(v/c)^2$, where L is the round-trip distance between A and B (and between A and C).

(d) A track-and-field runner experiences the same phenomenon when he or she runs round trips on a track on a windy day. The world record holder for the 400 m race (on a 400 m oval track) is 43.18 s by Michael Johnson of the US (1999). Estimate how much slower he is on a day with 2 mph winds than on a day without. (Approximate the oval track as a square with 100 m sides with the wind blowing along one of them if you wish.) The top two all-time records for the men's 800 m race are 1:41.11 by Wilson Kipketer of Denmark (1997) and 1:41.73 by Sebastian Coe of the UK (1981). Could Coe have had a better record than Kipketer if Kipketer had run on a day with 2 mph winds when Coe had not? (Correction: Actually, this wind analysis for the runners is incorrect because the medium they move relative to is the *ground*, not the air. A more correct analogy would be to consider the consequences of the ground moving like a treadmill at 2 mph, with each leg of the "square oval" having treadmills moving in the same direction, say West to East. Nevertheless, for the purposes of this problem, pretend that the runners are like airplanes and moving relative to the air instead. Some say that these Olympic athletes are "flying" anyway.)

Extra Credit Problems

6.4 IRFs in NIRFs, continued. Consider a physics demo performed in Regener Hall 103. (a) Use what you learned in problem 6.1 to estimate the precision (in both time and space) with which this demo can be well-described by an IRF completely contained within the room. (b) Estimate the maximum masses the objects in this demo can have such that they can be well-described by this IRF. Full credit for either part will only be awarded for estimates that agree with mine to within a factor of 100. (100 times larger or 100 times smaller.)

6.5 Principle of Relativity, continued. The Principle of Relativity states that the laws of physics are the same in all IRFs. IRFs can differ not just by relative velocities but by rotations and translations too. The rotation transformation between IRF coordinates is

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

for a rotation of the primed coordinates by the angle θ about the z axis relative to the unprimed coordinates. For definiteness in this problem, suppose Alice and Bob share overlapping IRFs, but Bob's coordinates are rotated by an angle θ about the z axis relative to Alice's.

(a) Use the rotation transformation to express the components of the velocity of an object in Bob's IRF in terms of the components of the velocity of the object as measured in Alice's IRF. (Hint: Use the chain rule from calculus.)

(b) Use the rotation transformation to express the components of the acceleration of an object in Bob's IRF in terms of the components of the acceleration of the object as measured in Alice's IRF.

(c) Use the rotation transformation to express the unit vector directions in Bob's IRF in terms of the unit vector directions as measured in Alice's IRF.

(d) Combine the results of parts (a)–(c) of this problem to show that $m\vec{\mathbf{a}} = m\vec{\mathbf{a}}'$, *i.e.*, Bob measures the same force on the object as Alice does.