UNM Physics 262, Problem Set 7, Fall 2006

Instructor: Dr. Landahl
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Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. **Please put your box number on your assignment, which is 952 plus your CPS number**, as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

7.1 Star Trekking.

"Warp factor two mister Sulu, and step on it!," exclaims Captain Kirk of the USS Enterprise. In the fictional TV series *Star Trek*, the USS Enterprise is capable of traveling at speeds greater than the speed of light. The speed "warp factor two" means travel at eight times the speed of light, according to Wikipedia. Knowing that the principle of causality and the constancy of the speed of light forbid anything from traveling faster than light speed, it is tempting to chalk up this whole warp speed nonsense to TV fiction. But is that right? In this problem we examine this question more carefully, and come to some surprising conclusions. In particular, we'll see that special relativity *does* allow one to travel to any spot in the universe in any amount of personal "wristwatch" (*i.e.*, *proper*) time.

(a) On its maiden voyage, the USS Enterprise leaves Earth at a speed of $0.8c$ and travels at this constant speed for 6 years of proper time (the time measured by the ship's chronometers). (i) How much time, in years, has passed on the Earth during the Enterprise's voyage? (ii) How much distance, in light-years, has the Enterprise traveled, as measured in the Earth's IRF? (iii) If you divide the distance from (ii) by the time from (i), you should get the velocity of the Enterprise, as measured in Earth's frame. On the other hand, if you divide the distance from (ii) by the time that has passed on the Enterprise, you will obtain a speed greater than the speed of light. What is that speed, as multiple of the speed of light? (iv) Why doesn't this result imply that the Enterprise is violating the principle of causality and the constancy of the speed of light?

(b) Suppose Sulu carries out Kirk's orders and travels at warp two relative to the IRF they started in. In other words, if one divides the Enterprise's distance traveled as measured in the original IRF by the proper time passed on the Enterprise, the speed is $8c$. (i) If they maintain this speed for three hours, how much time has passed in their original IRF? (ii) Warp 10 is supposed to be the maximum speed attainable by the USS Enterprise D, captained by Jean-Luc Picard on the TV sequel *Star Trek:The Next Generation.* According to Wikipedia, this speed is $1000c$. How far can a ship traveling at this warp factor go in 1 hour of proper time, using the same interpretation of what "warp factor" means?

(c) One of the devices on the Enterprise is a transporter. This device is supposed to convert an object to information, which is then sent at the speed of light ("beamed") to its final destination.[1] Evidently this device only works at short range. (Otherwise, why bother cruising around in a starship?) Suppose the Enterprise settles into a "geosynchronous" orbit around a planet having the same radius as the Earth, but with twice the rotational period and three times the mass. (i) How far is the ship from the center of the planet, in light-seconds? (ii) Suppose an exploration party is "beamed" to the surface via the transporter. How much do they age during their trip to the planet? (iii) The party looks around for an hour (as measured by their own wristwatches) and finds nothing to report. They then beam back to the ship. How much older are their crewmates on the ship than they are? Neglect any time dilation from gravity, which is a general relativistic effect that we haven't discussed in this class.

7.2 Speed Walking.

According to Merriam-Webster's Third New International Dictionary (unabridged)[2], to "walk" means to "go on foot without lifting one foot clear of the ground before the other touches the ground." In other words, at least one foot must be on the ground at all times. In this problem, we will use this definition to work out what the maximum speed of walking is as imposed by special relativity.

To begin with, consider a (*very!*) fast walker who moves her free foot forward at nearly the speed of light. Then one might argue (ambiguously) as follows: While the free foot is moving forward, the planted foot is on the ground, ready to be picked up *when* (red flag!) the free foot comes down in front. Half of the time each foot is in motion at nearly light speed and half of the time it is at rest. Therefore, the average speed of each foot, equal to the maximum possible speed of the walker, is half the speed of light.

Why is this argument ambiguous? Because of the relativity of simultaneity. The word *when* applied to separated events should set of warning bells in your head. The event "front foot down" (let's call it FrontDown) and the event "rear foot up" (let's call it RearUp) occur at different places along the line of motion. Observers in relative motion will disagree about whether or not the events FrontDown and RearUp occur at the same time. Therefore, they will disagree about whether or not the walker has one foot on the ground at all times in order to satisfy the dictionary definition of walking.

How can we remove the ambiguity in the definition of walking? One way is to make the conventional definition IRF-independent: One foot must be on the ground at all times *as measured in every IRF*. What limits does this place on the two events FrontDown and

[1]Believe it or not, such a device is allowed by the laws of quantum mechanics and *very* small scale demonstrations have recently been implemented in the lab. In the Oct. 5, 2006 issue of *Nature* (extremely recent!), such "quantum teleportation" between light and matter was reported for the first time (Sherson *et al.*, Nature **443**, 557). Teleportation between light and light had been done numerous times before, and teleportation between atoms was first done in 2004 (Riebe *et al.*, Nature **429**, 734; Barrett *et al.*, Nature **429**, 737). The original theory dates back to 1993 (Bennett *et al.*, Physical Review Letters **70**, 1895).

[2]This is the best American dictionary, period. Accept no substitutes.

RearUp? The rear foot must leave the ground after, or at least simultaneous with, the front foot touching the ground, as measured by all IRF observers. Use the following outline to derive the consequences of this definition for the maximum speed of walking.

(a) Consider the three possible relationships between events FrontDown and RearUp: timelike, lightlike, and spacelike. For each of these three relationships, write down answers to the following three questions

1. Will the temporal order of the two events be measured to be the same by all IRF observers?
2. Does this relationship adequately satisfy the frame-independent definition of walking?
3. If so, does this relationship give the maximum possible speed for walking?

Show that you answer “yes” to all three questions only for a lightlike relationship between the two events.

(b) A lightlike relationship between events FrontDown and RearUp means that light can just travel from one event to the other with no time left over. Let the distance between these events—the length of one step in the Earth frame—be the unit of distance and time. Show that for the limiting speed in this frame, each foot spends two units of time moving forward, then waits one unit while the light signal propagates to the other foot, then waits three units while the other foot goes through the same process. Summary: Out of six units of time, each foot moves forward at (nearly) the speed of light for two units. What is the average speed of each foot, and therefore the speed of the walker, as measured in the Earth frame?

(c) Draw a spacetime diagram for the Earth frame, showing worldlines for each of the walker’s feet and worldlines for the connecting light flashes. Add a worldline showing the averaged motion of her torso, always located halfway between the two feet in the Earth frame. Demonstrate that her torso moves at the speed of the walker reckoned above.

(d) After completing parts (a)–(c) of this problem, a student says, “We determined the maximum walking speed by finding a frame-independent definition of walking. Therefore this fast walker moves at the same speed as observed in every frame.” Is the student right? Why or why not?

7.3 The worst Thursday ever.

The book *The Hitchhiker’s Guide to the Galaxy* begins with the protagonist Arthur Dent waking up on “the worst Thursday ever,” looking out his window and seeing bulldozers. He learns that his home is to be razed that morning to make room for a new bypass. (“The plans were available in the city planning department for the past nine months,” the foreman informs him.) In a cosmic irony, a friend of Arthur’s by the name of Ford Prefect (who turns out to be an alien) informs him that the solar system is to be destroyed later that afternoon to make room for an intergalactic bypass. When the spaceship bulldozers arrive, they inform the entire planet by PA that the Earth will soon be obliterated in the name of progress. ("The plans were in the Alpha Centauri planning department for the past 50 years," the alien foreman informs the Earth.)

Let's pick up the story at this point and spin a tale of special relativity around it. Suppose the date and time of the story are different, so that the bulldozing aliens arrive at dusk on the first day of summer. The Sun is setting in the West and the planet Venus is visible in the same direction. On the opposite horizon the full Moon is rising and is due East. Suppose the bulldozing aliens arrive from the East from the star Proxima Centauri, which lies due East beyond the rising Moon. They say they have been traveling straight to Earth at a uniform velocity with a time stretch factor of $\gamma = 5/3$.

At the same instant that the aliens land, Arthur Dent sees the Sun explode. The aliens inform him that earlier, on their way to Earth, they shot a laser at the Sun, which caused this explosion. They warn him that the Sun's explosion emitted an immense pulse of particles moving at half the speed of light that will blow away Earth's atmosphere. In confirmation, shortly after the bulldozing aliens arrive, Arthur notices that the planet Venus, lying in the direction of the Sun, suddenly changes color.

Ford Prefect offers Arthur a ride on his spaceship. The ship will take off in exactly 7 minutes after the bulldozing aliens' arrival at Earth. It will flee in an easterly direction away from the Sun at the top speed of his ship, which has a time stretch factor of $\gamma = 25/7$.

(a) Do Arthur and Ford make it?

Draw a detailed Earth spacetime diagram showing the events and worldlines of this story. Use the following information

- The Sun is 8 light minutes from the Earth.
- Venus is 2 light minutes from the Earth.
- Assume that the Sun, Venus, the Earth, and the Moon all lie along a single direction in space and are relatively at rest during this short story. The incoming and outgoing paths of the alien ships lie along the same line in space.
- All takeoffs and landings involve instantaneous changes from initial to final speed.
- $5^2 - 3^2 = 4^2$ and $25^2 - 7^2 = 24^2$.

(b) Plot the ***events*** below on your diagram, labeled with the following ***numbers***:

1 Arthur's location when the aliens arrive at Earth (at the origin).

2 The Sun explodes.

3 Light from the Sun explosion reaches Arthur.

4 Venus' atmosphere is blown away.

5 Light from event 4 reaches Arthur.

6 Arthur and Ford depart Earth (they hope).

7 The Earth's atmosphere is blown away.

(c) Plot the ***worldlines*** below on your diagram, labeled with the following ***capital letters***:

A Arthur's worldline.

B The worldline of the Earth.

C The worldline of the bulldozing aliens.

D The worldline of the Sun.

E The worldline of Venus.

F The worldline of light from the Sun's explosion.

G The worldline of the "speed-one-half" pulse of particles from the Sun's explosion.

H The worldline of light emitted when Venus loses its atmosphere.

I The terminal part of the worldline of the laser cannon pulse fired at the Sun by the aliens.

(d) Write numerical values for the speed v (as a fraction of c) on every segment of all worldlines.

7.4 Look ma, no coordinates!.

Although we have focused on coordinates and coordinate transformations in our discussion of IRFs, it is not necessary to have such a coordinate system to have an IRF. All that one needs are reference events; from these one can generate a "spacetime map" of the events without any recourse to coordinates. The same thing holds true for ordinary "space maps." In this problem, we go through examples of how this works in both cases.

(a) The table below shows distances between cities. The units are kilometers. Assume all cities lie on the same flat plane.

Dist. to city	A	B	C	D	E	F	G	H
From city								
A	0	20.0	28.3	28.3	28.3	20.0	28.3	44.7
B		0	20.0	20.0	44.7	40.0	44.7	40.0
C			0	40.0	40.0	44.7	56.6	60.0
D				0	56.6	44.7	40.0	20.0
E					0	20.0	40.0	72.1
F						0	20.0	56.6
G							0	44.7
H								0

(i) Use a ruler and a compass (the kind of compass that makes circles) to construct a map of these cities. Choose a convenient scale, such as one centimeter on the map corresponds to ten kilometers on Earth. [How to start? First pick any city to be the center of the map. Then choose any second city to be "due North"—that is, along any arbitrary direction you select. After these two choices, you still have two choices for where to put the third city; choose one. The map should fall into place as you continue.]

(ii) If you rotate your completed map in its own plane—for example, by turning it while keeping it flat on the table—does the resulting map also satisfy the distance entries above?

(iii) Hold up your map between you and a light, with the marks on the side of the paper facing the light. Does the map you see from the back also satisfy the table entries?

(b) The table below shows timelike intervals between events in spacetime. The units are meters. The events occur in the time sequence $ABCD$ in all frames and along a single line in space in all frames. (They do *not* occur along a single line on the spacetime map.)

Interval to event	A	B	C	D
From event				
A	0	1.0	3.161	5.196
B		0	2.0	4.0
C			0	2.0
D				0

(i) Use a ruler and the hyperbola graph below to construct a spacetime map of these events. Draw this map on thin paper so you can lay it over the hyperbola graph and see the hyperbolas. As in part (a), choose a convenient scale, such as one unit on the map corresponds to one meter of timelike interval. [How to start? First pick event A to be the origin of the map. Then choose event B to occur at the same place as event A. That is, event B is located on the positive time axis with respect to event A. After plotting B, use your ruler to draw this straight time axis through events A and B. Keep this line parallel to the vertical lines on the hyperbola graph in all later constructions. After these two choices, you still have two choices for the spacetime location for event C. Then go on to plot event D.

(ii) Now take a new piece of paper and draw a spacetime map for another reference frame. Choose event D to be at the origin of the spacetime map. This means that all other events occur before D. Hence turn the hyperbola plot upside down, so that the hyperbolas open downward. Choose event B to occur at the same place as D. Now find the locations of A and C using the same strategy as in part (i).

(iii) Find an approximate value for the relative speed of the two frames for which you have made spacetime plots.

(iv) Hold one of your spacetime maps up to the light with the marks on the side of the paper facing the light. Does the map you see from the back also satisfy the table entries?

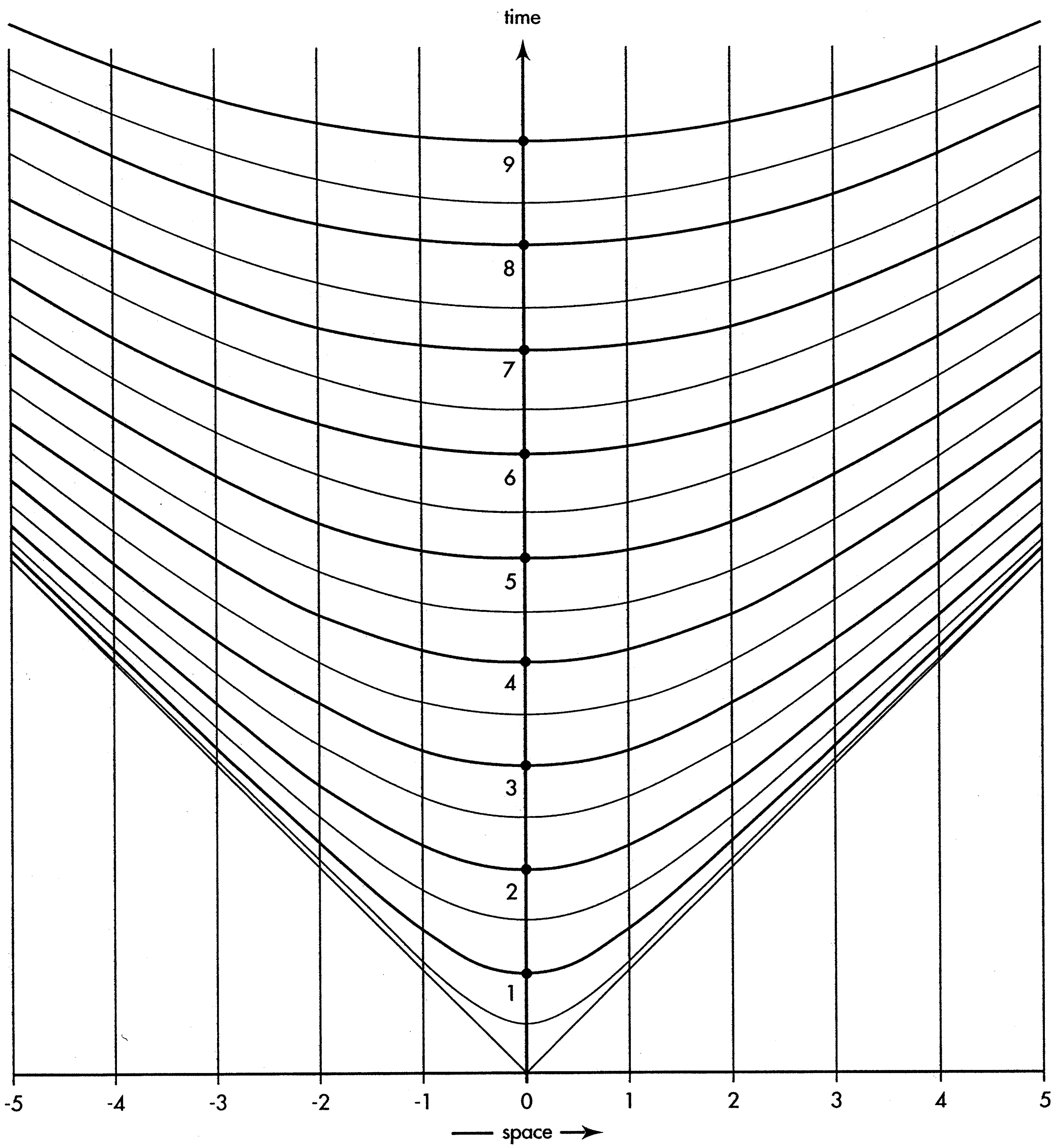

Figure 1: Template of hyperbolas for converting intervals into a spacetime map.