UNM Physics 262, Problem Set 8, Fall 2006

Instructor: Dr. Landahl
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Due: October 25, 2006

Do all of the exercises and problems listed below. Hand in your problem set in the rolling cart hand-in box, either before class or after class, or in the box in the Physics and Astronomy main office by 5 p.m. **Please put your box number on your assignment, which is 952 plus your CPS number**, as well as the course number (Physics 262). Show all your work, write clearly, indicate directions for all vectors, and be sure to include the units! Credit will be awarded for clear explanations as much, if not more so, than numerical answers. Avoid the temptation to simply write down an equation and move symbols around or plug in numbers. Explain what you are doing, draw pictures, and check your results using common sense, limits, and/or dimensional analysis.

8.1 The Millennium Falcon

Han Solo and Chewbacca, intrepid pilots of the *Millennium Falcon* spaceship in the fictional *Star Wars* movies, are on the lam as usual from bounty hunters. While hanging out at one of their favorite haunts, the Mos Eisley Cantina on the planet Tatooine, they are spotted by the bounty hunter Greedo. In the original *Star Wars* movie, Han shoots Greedo dead with his pistol (a "blaster") from under the table. In this problem, we will consider an alternative storyline with some relativistic twists. (Hint: Every part of this problem uses the relativistic velocity transformation formulas we discussed in class.)

(a) After being spotted by Greedo, Han and Chewie instead board the *Falcon* and high tail it out of there at a proper velocity of $w_1 = c$ relative to the Tatooine IRF, where c is the speed of light. They soon discover that Greedo is hot on their trail and gaining on them in his own spaceship. Han employs his self-named "Solo Boomerang" maneuver in which he reverses course quickly in the hopes that Greedo won't realize where the *Falcon* went until it's too late. After the maneuver, the proper velocity of the *Falcon* is $w_2 = -\sqrt{8}c$ relative to the IRF they were just in. (i) What are the coordinate velocities v_1 and v_2 corresponding to the proper velocities w_1 and w_2? (ii) What is the final coordinate velocity of the *Millennium Falcon*, as measured in the Tatooine IRF? (iii) What is the final proper velocity of the *Falcon*, as measured in the Tatooine IRF?

(b) Greedo was too clever for this maneuver to outwit him, and he's soon on their trail again. Chewie spots a gaseous nebula that they might be able to hide in. They approaching the nebula with a coordinate velocity v. The index of refraction of light in the nebula is n, so the speed of light in the nebula is not c but instead c/n, as measured in the rest frame of the nebula. Chewie fires a laser into the nebula and uses the ship's computers to measure the speed of the laser pulse in the nebula. He's hoping that the computer will tell him that the index of refraction of the nebula is one that will enable them to successfully hide from Greedo. (i) Develop an expression relating what the speed u that the computer measures is in terms of c, n, and v. Chewbacca will need to solve this equation for n to answer his question. (But you don't have to.) (*ii*) Show that for $v \ll c$, you can expand this expression in powers of v/c using a Taylor expansion (which in this case is the binomial expansion) to yield the result[1]

$$u \approx \frac{c}{n} - v\left(1 - \frac{1}{n^2}\right).$$

(*c*) Greedo approaches the nebula and suspects Han and Chewie are hiding inside it. Impatient to wait until he arrives at the nebula where he can slow down to a nonrelativistic speed to do a thorough scan, Greedo sends out a spotlight beam along his direction of motion such that the angle the beam makes with the x'-axis is measured to be ϕ' in Greedo's IRF. (*i*) Show that the angle ϕ the beam makes with the x-axis as measured in the nebula's IRF is

$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'},$$

where v is the coordinate velocity with which Greedo is moving towards the nebula.

(*ii*) Show that if Greedo had simply emitted a light pulse uniformly in all directions, the 50 percent of the light that goes into the forward hemisphere is concentrated into a narrow forward cone of half-angle ϕ_0 as measured in the nebula frame, where ϕ_0 obeys the equation

$$\cos\phi_0 = v/c.$$

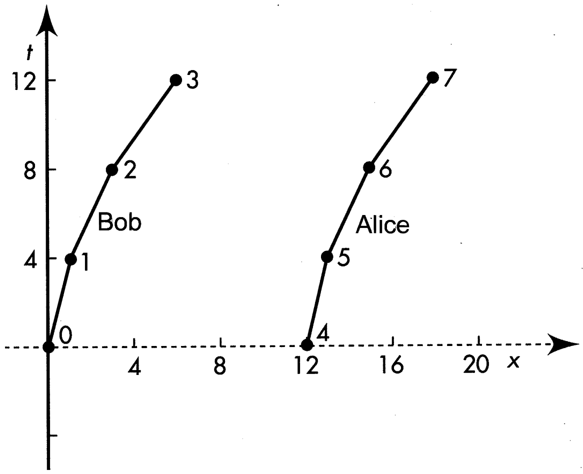
This curious phenomenon is called the *headlight effect* in special relativity.

8.2 The identically accelerated twins paradox.

Alice and Bob ("twins" in this problem) own identical spaceships each containing the same amount of fuel. Alice's ship is initially positioned a distance to the right of Bob's in the Earth's IRF. On their twentieth birthday they blast off at the same instant in the Earth frame and undergo identical accelerations to the right as measured by their friend Charlie, who remains at home on Earth. Charlie further observes that the twins run out of fuel at the same time and move thereafter at the same speed v. Charlie also measures the distance between Alice and Bob to be the same at the end of the trip as at the beginning.

Alice and Bob compare the ships' logs of their accelerations and find the entries to be identical. However, when both have stopped accelerating, Alice and Bob, in their new IRF, discover that Alice is older than Bob! How can this be, since they have an identical history of accelerations?

[1] This formula was important in the history of relativity; before Einstein came along, this effect had been experimentally observed and was attributed to "ether dragging."



Earth Frame Observations

Event number	x-position (light years)	Time (years)
0	0	0
1	1	4
2	3	8
3	6	12
4	12	0
5	13	4
6	15	8
7	18	12

EARTH FRAME

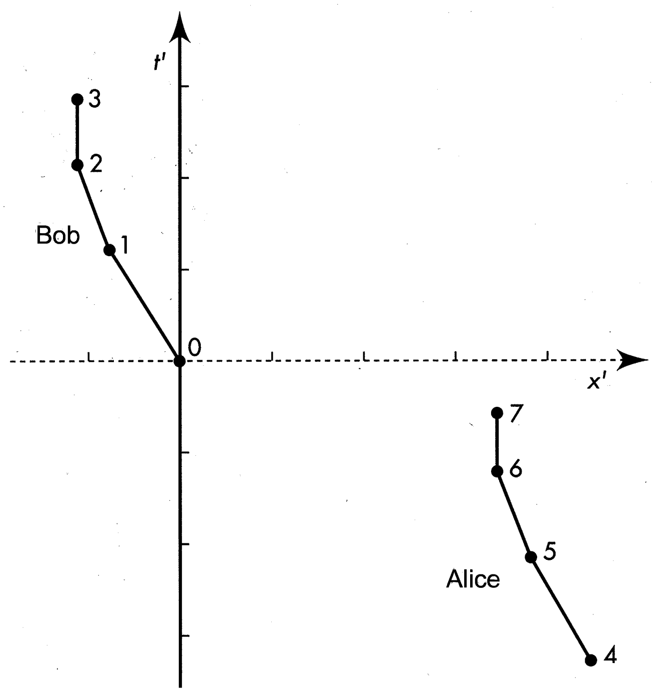


Figure 1: Top: Worldlines of Alice and Bob as observed in the Earth frame of Charlie. Bottom: Worldlines of Alice and Bob as observed in the "final" IRF in which Alice and Bob come to rest after burnout.

(a) Analyze a simpler trip, in which each spaceship increases speed not continuously but by impulses, as shown in the first spacetime diagram and the event table. (i) How far apart are Alice and Bob at the beginning of their trip, as observed in the Earth frame? (ii) How far apart are they at the end of their accelerations? (iii) What is the final speed v (not the average speed) of the two spaceships? (iv) How much does each astronaut age along the worldline shown in the diagram? (The answer is not the Earth time of 12 years.)

(b) The second spacetime diagram shows the two worldlines as recorded in a rocket frame moving with the final velocity of the two astronauts. (i) Copy the figure. On your copy, extend the worldlines of Alice and Bob after each has stopped accelerating. Label your figure to show that Alice stopped accelerating before Bob as observed in this frame. (ii) Will Bob age the same between events 0 and 3 in this frame as he aged in the Earth frame? (iii) Will Alice age the same between events 4 and 7 in this frame as she aged in the Earth frame?

(c) Now use the Lorentz transformation to find the space and time coordinates of one or two critical events in this final rest frame of the twins in order to answer the following questions:

(1) How many years earlier than Bob did Alice stop accelerating?

(2) What is Bob's age at event 3? (Not the rocket time t' of this event!)

(3) What is Alice's age at event 7?

(4) What is Alice's age at the same time (in this frame) as event 3?

(5) What are the ages of Alice and Bob 20 years after event 3, assuming that neither moves again with respect to this frame?

(6) How far apart in space are Alice and Bob when both have stopped accelerating?

(7) Compare this separation with their initial (and final!) separation measured by Charlie in the Earth frame.

(d) Extend your results to the general case in which Charlie on Earth observes a period of identical *continuous* accelerations of the two twins.

(1) At the two start-acceleration events (the two events at which the twins start their rockets), the twins are the same age as observed in the Earth frame. Are they the same age at these events as observed in every IRF?

(2) At the two stop-acceleration events (the two events at which the rockets run out of fuel), are the twins the same age as observed in the Earth frame? Are they the same age at these events as observed in every rocket frame?

(3) The two stop-acceleration events are simultaneous in the Earth frame. Are they simultaneous as observed in every IRF? (No!) Whose stop acceleration event occurs first as observed in the final frame in which both twins come to rest? (Recall Einstein's Train Paradox that we talked about in class.)

(4) "When Bob stops accelerating„ Alice is older than Bob." Is this statement true according to the astronauts in their final rest frame? Is the statement true according to Charlie in the Earth frame?

(5) Criticize the lack of clarity (swindle?) of the word *when* in the statement of the problem: "However, when both have stopped accelerating, Alice and Bob, in their new test frame, discover that Alice is older than Bob."

(e) Suppose that Alice and Bob both accelerate to the left, so that Bob is in front of Alice, but their history is otherwise the same. Describe the outcome of this trip and compare it with the outcomes of the original trip.

(f) Suppose that Alice and Bob both accelerate in a direction perpendicular to the direction of their separation. Describe the outcome of this trip and compare it with the original trip.

8.3 Doppler mania.

(a) A UNM physics major is arrested for going through a red light. Having just taken Physics 262, she devises a cunning defense using relativity. In court, she pleads that she approached the intersection at such a speed that the red light looked green to her due to the Doppler effect.[2] Unfortunately for her, the judge was also a graduate of Physics 262, and he knows all about the relativistic Doppler effect. He changes the charge to speeding and fines her one dollar for every mile per hour that she exceeded the local speed limit of 35 mph. (i) What is the fine? Take the wavelength of green light to be 530 nm and the wavelength of red light to be 650 nm. Notice that the light propagates in the negative x-direction.

(ii) This hefty fine wasn't enough to deter her from running another red light only a month later. This time, she is brought before a different judge. She figures that it is unlikely that this judge has also has taken Physics 262. However, she is wary of making an identical defense because the judge might just look at her previous record to determine what the fine should be. No need to pay *that* again! This time, she argues that once again the Doppler effect is to blame, but with a different twist. For this incident, she argues, she was approaching the intersection at such a speed that the red light had shifted beyond the limits of human vision. She argues that the light appeared to not be working, so she ignored it. Her plan backfires, as this judge was also graduate of Physics 262, much to her dismay. Rather than fining her for not stopping at a powered-out stoplight, which she should have done, he decides to teach her a lesson by fining her once again one dollar for every mile per hour that she exceeded the local speed limit of 35 mph. Taking the limit of human vision to be ultraviolet light of wavelength 380 nm, how much is this second fine? The moral of the story: Crime doesn't pay!

[2] This defense ignores the fact that red and green lights are at different locations on a stoplight, which she should have used to determine what the signal was regardless of what color she saw. Color-blind people have to do this every time they drive!

(*b*) To increase the safety of space travel in the distant future, red beacons are placed along the edges of "space lanes" marking designated routes for interstellar travel. Each beacon is essentially a lightbulb that emits spectrally pure red light uniformly in all directions in its rest frame. Consider a space traveler who approaches one of these bulbs at a speed close to the speed of light along a straight-line path whose perpendicular distance to the bulb is b. (Remember, transverse lengths do not contract in special relativity, so b is a measurement that is agreed upon in all IRFs moving in this direction.) Both the color and the intensity of the light that reach the traveler from the beacon vary with the traveler's proper time. Describe these changes qualitatively at several stages as the light bulb passes the traveler. Consider both the Doppler shift and the headlight effect (See problem 8.1). How does what you describe change as b varies? How does it change as the traveler's speed v with respect to the beacon varies?

(*c*) One of the great discoveries of the 20th century was that the universe is expanding. In 1929, American physicists Edwin Hubble and Milton Humason[3] discovered that the further away a galaxy is from us, the faster it appears to be receding. The relation between the velocity of a galaxy, v, and its distance from us, r, is $v = Hr$, now known as Hubble's law. The constant H is called the Hubble constant and has the value $H = 70$ (km/s)/Mpc, where a Mpc is a "megaparsec." Look up on the Internet or elsewhere how a parsec is defined and what value it has in standard SI units.

(*i*) Write H in SI units.

A clever way of measuring the velocity of a receding galaxy is through the Doppler effect. The atoms in the galaxy emit light at very specific wavelengths as measured in their own rest frame. (These wavelengths can be calculated using Quantum Mechanics, which we will soon study.) However, because they are receding from us, their frequency and wavelengths are measured to be shifted. The *redshift* of a galaxy is defined as

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}}.$$

(*ii*) A spectral line of wavelength 730 nm is identified to be one of the lines of hydrogen in a distant galaxy that, for hydrogen in the laboratory, has the wavelength 487 nm. How fast is the observed galaxy moving relative to the Earth, in m/s? What is its redshift?

(*iii*) Combine the definition of redshift, the relativistic Doppler effect, and Hubble's law to answer the following questions in SI units. How far is an object with a redshift of $z = 0.5$ from Earth? How far is an object of $z = 4$ from Earth? The cosmic microwave background radiation (CMB) is about 13.7 billion light years away from us. What is its redshift?

[3]Humason started out as a janitor at the Mount Wilson Observatory; even in astrophysics it is possible to climb the career ladder!