Ph262 PS 9

OH 4.10 (a) For 100 kg @ 0.5 c, the kinetic energy is

$$KE = \frac{mc^2}{\sqrt{1-v^2/c^2}} = \frac{(100\,kg)(3\times10^8\,m/s)^2}{\sqrt{1-0.5^2}} = \boxed{1.4\times10^{18}\,J}$$

Compared to the yearly US expenditure of 10^{20} J, this is

$\boxed{\text{1.4 \% of the yearly US energy expenditure.}}$

(b) For 1g @ 0.5 c, the kinetic energy is 10^{-5} what it was in part (a):

$$\boxed{KE = 1.4\times10^{13}\,J = 1.4\times10^{-5}\,\%\ \text{US energy expenditure}}$$

OH 4.11 For an energy power of 3.9×10^{26} W, the mass loss over a year is

$$\Delta M = \frac{dM}{dt}\Delta t = \frac{3.9\times10^{26}\,W}{(3\times10^8\,m/s)^2}(3.16\times10^7\,s) = \boxed{1.4\times10^{17}\,kg}$$

OH 4.12 (a) For a 10 kton TNT explosion, using 1 kton $= 4.2\times10^{12}$ J, the amount of rest mass converted to energy is

$$M = E/c^2 = \frac{10\,(4.2\times10^{12}\,J)}{(3\times10^8\,m/s)^2} = \boxed{4.7\times10^{-4}\,kg}$$

(b) The total system mass is unchanged in the explosion, because the energy of the system possesses mass.

Ph 262 PS 9

OH 4.14 For $B = 10^3\,T$, the energy density is

$$E = \frac{B^2}{2\mu_0} = \frac{(10^3 T)^2}{2(1.26\times10^{-6} H/m)} = \boxed{4.0\times10^{11}\,J/m^3}$$

The associated mass density is

$$m = E/c^2 = \frac{4.0\times10^{11}\,J/m^3}{(3\times10^8\,m/s)^2} = \boxed{4.4\times10^{-6}\,kg/m^3}$$

OH 4.23 Using the transformation equation

$$\begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \gamma \begin{bmatrix} 1 & -V/c \\ -V/c & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \text{for } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} ct \\ x \end{bmatrix} \text{ and } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} E/c \\ p \end{bmatrix}$$

We have

$$\vec{p}'\cdot\vec{x}' = E't' - p_x'x' - p_y'y' - p_z'z'$$

$$= \gamma^2(E/c - \tfrac{V}{c}p_x)(ct - \tfrac{V}{c}x) - \gamma^2(-\tfrac{V}{c}\tfrac{E}{c} + p_x)(-\tfrac{V}{c}ct + x) - p_y y - p_z z$$

$$= Et\,\underbrace{\cancel{\gamma^2(1-\tfrac{V^2}{c^2})}}_{1} - p_x x\,\underbrace{\cancel{\gamma^2(1-\tfrac{V^2}{c^2})}}_{1} - p_y y - p_z z$$

$$+ \gamma^2\left[\cancel{-\tfrac{E}{c}\tfrac{V}{c}x} - \cancel{\tfrac{V}{c}p_x ct} + \cancel{\tfrac{E}{c}\tfrac{V}{c}x} + \cancel{\tfrac{V}{c}p_x ct}\right]$$

$$= Et - p_x x - p_y y - p_z z$$

$$= \vec{p}\cdot\vec{x} \implies \boxed{\vec{p}\cdot\vec{x} = \vec{p}'\cdot\vec{x}'}$$