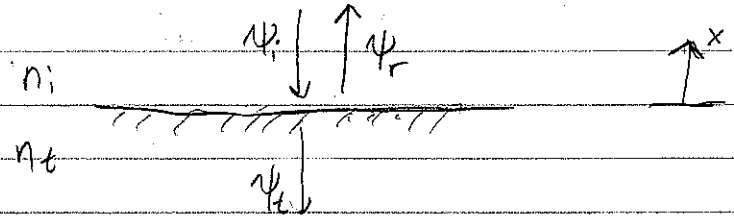
Phaed Lecture 10

Ch 35 **Wave Optics**

$\psi = \vec{E} \cos(kx - \omega t + \phi)$ (amplitude, wave number, angular freq, phase)

$k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda}$ $\quad \omega = \frac{2\pi}{T}$ $\quad c = \frac{\omega}{k}$

Reflection & Transmission at Normal incidence (At angles requires Maxwell's Eqns - Polarization)



$\psi_i = E_i \cos(k_i x - \omega_i t)$

$\psi_t = E_t \cos(k_t x - \omega_t t)$

$\psi_r = E_r \cos(k_r x + \omega_r t)$

$\omega_i = \omega_t = \omega_r \equiv \omega$

$k_i = \frac{2\pi}{\lambda_i} = \frac{2\pi n_i}{\lambda_i} = k_r$

$k_t = \frac{2\pi n_t}{\lambda_t}$

Continuity at surface: $\psi_i + \psi_r = \psi_t$ at $x = 0$

(I) $\boxed{E_i + E_r = E_t}$

Ph 262 Lecture 10

Conservation of Energy:

$\vec{S}_i = \vec{S}_r + \vec{S}_t$ at $x=0$ (Poynting vectors)

$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

$|\vec{B}| = E/c$ in free space

$= E/v = nE/c$ in a medium

$\Rightarrow |\vec{S}| = \frac{n}{\mu_0} E^2$

Hence

$$n_i E_i^2 = n_i E_r^2 + n_t E_t^2$$

Reflection coefficient: (Amplitude)

$r \equiv \psi_r/\psi_i$ at $x=0$

$t \equiv \psi_t/\psi_i$ at $x=0$

Boxed equations (boundary conditions) in these variables are now

(I): $1 + r = t$

(II): $1 = r^2 + \frac{n_t}{n_i} t^2 \quad \Leftrightarrow (1-r)(1+r) = \frac{n_t}{n_i} t^2$

Ph262 Lecture 10

Combining (I) & (II):

$$(1-r)t = \frac{n_t}{n_i} t$$

$$t = \frac{n_i}{n_t}(1-r)$$

$$\Rightarrow 1 + r = \frac{n_i}{n_t}(1-r)$$

$$r\left(1 + \frac{n_i}{n_t}\right) = \frac{n_i}{n_t} - 1$$

$$r = \frac{n_i - n_t}{n_i + n_t}$$

Reflection coefficient (Intensity)

$$R \equiv r^2 = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2$$

Example: air → glass

$$R = \left(\frac{1-1.5}{1+1.5}\right)^2 = \left(\frac{-1/2}{5/2}\right)^2 = \frac{1}{25} = 4\%$$

⇒ 4% transmission loss every time light (of any wavelength) travels from air into glass! (or vice-versa)

8% for a pane of glass! $\left.\begin{array}{r} \downarrow \\ -4\% \\ -4\% \end{array}\right\} -8\%$

Aside: metals have imaginary n's, so $R = 1$ for them. (You can verify this at home.)

Ph26a Lecture 10

π phase shifts upon reflection:

Air → glass: $r < 0$

Glass → air: $r > 0$

Air → glass: $|E_r| = -|rE_i|$

$A\cos(\theta + \pi) = A\cos\theta \cancel{\cos\pi}^{-1} - A\sin\theta \cancel{\sin\pi}^{0}$

$= -A\cos\theta$

Amplitude sign flip = π phase shift

Do Multiple reflections next.

Interference: When waves add at fixed place

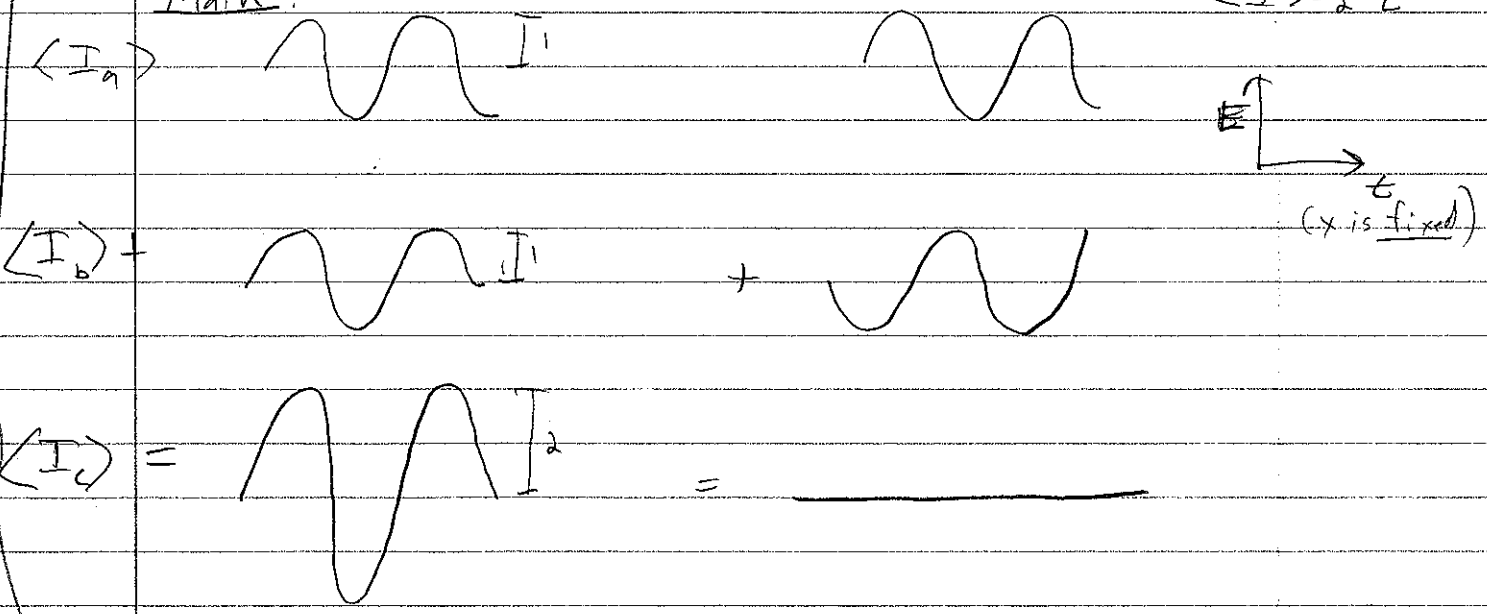
Diffraction: When many waves add at fixed place

Physics: Where do waves come from?

Math:

Skip

$\langle I \rangle = \frac{1}{2} E^2$



Constructive Interference

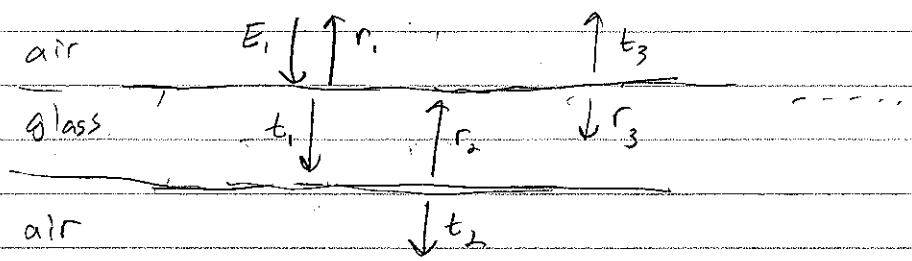
$\frac{1}{2}(2)^2 > \frac{1}{2} + \frac{1}{2}$

Destructive interference

$\frac{1}{2}(0) < \frac{1}{2} + \frac{1}{2}$

Ph262 Lecture 10

Multiple reflections



Must add waves : Interference