Wed 11/7/07, 7pm: Wiki Tutorial
-8:30

Pizza @ 6pm

Correction next time

$\mathcal{E}(\rho)$

$r_2 \to \underline{\underline{p}} + (1-p) r_2$

$\boxed{\text{Ph 452}}$ Lecture 20

QI Quick Review

- $\rho \to U \rho U^\dagger$, $\rho \to \frac{\Pi_k \rho \Pi_k}{tr(\Pi_k \rho)}$, $prob(k) = tr(\Pi_k \rho)$

- Quantum Operation OSR:

$$\rho \to \sum A_k \rho A_k^\dagger, \quad \sum A_k^\dagger A_k = I$$

- Motivation 1: "Church of the larger Hilbert space" ("Dilation")

$$\rho_A \to tr_B(U_{AB}\, \rho_A \otimes \sigma_B\, U_{AB}^\dagger) \qquad A_k = {}_B\langle k| U_{AB} |\psi\rangle_B$$

$|\psi\rangle_B {}_B\langle\psi|$

- Motivation 2: Axiomatic

All linear CPTP maps have an OSR

- OSR subsumes unitary, unconditional measurement evolution.

- OSR ambiguity:

$$\mathcal{A}(\rho) = \mathcal{B}(\rho) \iff A_i = \sum u_{ij} B_j$$

Proof: NC Thm 8.2

$\boxed{\text{Ph 432}}$ Lecture 2d

Example Channels:

Bit-flip channel $\mathcal{B}(p)$

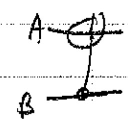
$$\rho \to (1-p)\,\rho + p\,X\rho X \qquad \begin{array}{l} A_0 = \sqrt{1-p}\, I \\ A_1 = \sqrt{p}\, X \end{array}$$

Unitary rep:

$$A_0 = {}_B\langle 0|\, \Lambda_{BA}(X) \left(\sqrt{1-p}\,|0\rangle_B + \sqrt{p}\,|1\rangle_B\right) = \sqrt{1-p}\; I$$

$$A_1 = {}_B\langle 1|\, \Lambda_{BA}(X) \left(\sqrt{1-p}\,|0\rangle_B + \sqrt{p}\,|1\rangle_B\right) = \sqrt{p}\; X$$

$\Lambda_{BA}(X) =$ controlled-X = CNOT =



$$= \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \end{pmatrix} \end{array}$$

Bloch sphere:

$$\rho = \tfrac{1}{2}\left(I + \vec{r}\cdot\vec{\sigma}\right) \qquad 0 \le |\vec{r}| \le 1$$

X Y Z

$XY = iZ$

iZX

$= i\,iY$

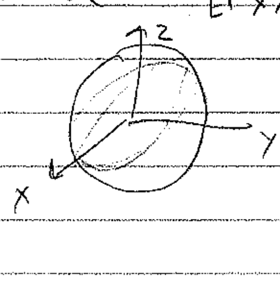
$= -Y$

$XZ = -iY$

$-iYX$

Factoid: $XXX = X$, $XYX = -Y$, $XZX = -Z$

$$\rho \to \tfrac{1}{2}\left(I + \left[r_x, (1-2p)\,r_y, (1-2p)\,r_z\right]\cdot\vec{\sigma}\right)$$



shrinks to cigar-like ellipsoid along x-axis.

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Process is <u>irreversible!</u>

$$\mathcal{B}^{-1}\left(\tfrac{1}{2}(I \pm (1-2p)Y)\right) = \tfrac{1}{2}(I \pm Y)$$

$$\tfrac{1}{2}\mathcal{B}^{-1}(I) \pm \tfrac{1}{2}(1-2p)\mathcal{B}(Y) = \tfrac{1}{2}(I \pm Y) \qquad \text{by linearity}$$

$$\Rightarrow \mathcal{B}^{-1}(Y) = \frac{1}{1-2p}Y, \quad \mathcal{B}^{-1}(I) = I$$

$$\Rightarrow \mathcal{B}^{-1}\left(\tfrac{1}{2}(I + Y)\right) = \underbrace{\tfrac{1}{2}\left(I + \frac{1}{1-2p}Y\right)}_{\text{Not a valid density matrix for } p>0!}$$

Is physical, though, with prior entanglement

$\Rightarrow \mathcal{B}^{-1}$ is not a quantum operation.

<u>Phase-flip channel</u> $\mathcal{P}(p)$

$$\rho \to (1-p)\rho + pZ\rho Z$$

Unitary rep: Same as BFC but using $\Lambda(Z)$

Bloch sphere: ellipsoid along z-axis. $[r_x, r_y, r_z] \Rightarrow [(1-2p)r_x, (1-2p)r_y, r_z]$

<u>Amplitude-damping channel</u> $\mathcal{A}(p)$

Spontaneous decay of an atom.

Unitary rep:

$$|0\rangle|0\rangle_B \to |0\rangle|0\rangle_B$$
$$|1\rangle|0\rangle_B \to \sqrt{1-p}\,|1\rangle|0\rangle_B + \sqrt{p}\,|0\rangle|1\rangle_B$$

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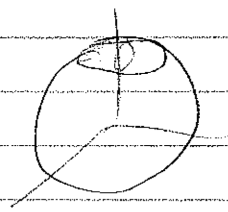
OSR:

$$A_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix}$$

Bloch sphere:

important

$$(r_x, r_y, r_z) \to (\sqrt{1-p}\, r_x,\ \sqrt{1-p}\, r_y,\ (1-p) r_z + p)$$



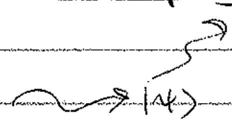
Sphere shrinks & squashes towards North pole.

Physics: $p = 1 - e^{-t/T_1}$, T_1 is a "decay time"

Note $\mathcal{E}(I) = \begin{pmatrix} 1+p & 0 \\ 0 & 1-p \end{pmatrix} \neq I$. $(= I + pZ)$

A <u>nonunital</u> channel

<u>Phase-damping channel:</u>



State-dependent scattering off of state.

$$|0\rangle|0\rangle_B \to |0\rangle(\sqrt{1-p}\,|0\rangle + \sqrt{p}\,|1\rangle)$$
$$|1\rangle|0\rangle_B \to |1\rangle(\sqrt{1-p}\,|0\rangle + \sqrt{p}\,|2\rangle)$$

OSR:

$$A_0 = \sqrt{1-p}\, I, \quad A_1 = \begin{bmatrix} \sqrt{p} & 0 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{bmatrix}$$

Bloch sphere:

$$(r_x, r_y, r_z) \to ((1-p) r_x, (1-p) r_y, r_z)$$

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Looks the same as

$$B_0 = \sqrt{1-\tfrac{p}{2}}\, I, \quad B_1 = \sqrt{\tfrac{p}{2}}\, Z \qquad \text{(Phase-flip channel) (scaled)}$$

Relationship:

$$\begin{array}{cccc} & I & Z & \\ U = & \left[\begin{matrix} \sqrt{\frac{2(1-p)}{2-p}} & \sqrt{\frac{p}{2(2-p)}} & \sqrt{\frac{p}{2(2-p)}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\sqrt{\frac{p}{2-p}} & \sqrt{\frac{1-p}{2-p}} & \sqrt{\frac{1-p}{2-p}} \end{matrix}\right] & & \end{array} \qquad B_i = \sum u_{ij} A_j$$

How to obtain:

$$A_0 = \sqrt{1-p}\, I, \quad A_1 = \frac{\sqrt{p}}{2}(I+Z), \quad A_2 = \frac{\sqrt{p}}{2}(I-Z)$$

Row 1:

$$B_0 = \sqrt{1-\tfrac{p}{2}}\, I = u_{00}\sqrt{1-p}\, I + u_{01}\frac{\sqrt{p}}{2}(I+Z) + u_{02}\frac{\sqrt{p}}{2}(I-Z)$$

$$\Rightarrow \sqrt{1-\tfrac{p}{2}} = u_{00}\sqrt{1-p} + u_{01}\frac{\sqrt{p}}{2} + u_{02}\frac{\sqrt{p}}{2}$$

$$0 = u_{01}\frac{\sqrt{p}}{2} + u_{02}\frac{\sqrt{p}}{2}$$

Also do $\frac{1}{2}(I \pm Z)$ vs I, Z

$$U^{\dagger}U = I \Rightarrow |u_{00}|^2 + |u_{01}|^2 + |u_{02}|^2 = 1$$

Row 2:

$$B_1 = \sqrt{\tfrac{p}{2}}\, Z = u_{10}A_0 + u_{11}A_1 + u_{12}A_2$$

$$\Rightarrow u_{11} = -u_{12}$$

$$0 = \sqrt{1-p}\, u_{10} + 2\sqrt{\tfrac{p}{2}}\, u_{11}$$

$$|u_{10}|^2 + |u_{11}|^2 + |u_{12}|^2 = 1$$

Finally, columns of U sum to 1, rows and cols $\perp$

Choose phases so that $\langle \text{row } 1 | \text{row } 2 \rangle = 0$

Row 3:

$\| |\text{col } j\rangle \| = 1, \quad \langle \text{col } i | \text{col } j \rangle = 0 \rightarrow$ choose phases appropriately.

Physics: $p = 1 - e^{-t/T_2}$, T_2 is "spin-spin" relaxation time

$\boxed{\text{Ph 462}}$ Lecture 20

Depolarizing Channel:

$$\rho \to (1-p)\,\rho + \frac{p}{3}\left(X\rho X + Y\rho Y + Z\rho Z\right)$$

Factoid: $\frac{1}{2}\left(I\rho I + X\rho X + Y\rho Y + Z\rho Z\right) = I$

$$\Rightarrow \quad X\rho X + Y\rho Y + Z\rho Z = 2I - \rho$$

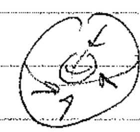
$$\rho \to \left(1 - \frac{4}{3}p\right)\rho + \frac{4}{3}p\left(\frac{I}{2}\right)$$

$$= \left(1 - \frac{4}{3}p\right)\rho + \frac{4}{3}p \int_{SU(2)} dU\, U\rho U^{\dagger}$$

("For those of you who know some calculus")

Bloch sphere:

$$\vec{r} \to (1-p)\,\vec{r}$$



Next time:
POVMs