$\boxed{\text{Ph 452}}$ Lecture 28

Quantum Communication

"Source coding" ① How much can a quantum source $X = \{\rho_x, p_x\}$ be compressed?

"channel coding" ② At what rate can one communicate reliably over a quantum channel $\mathcal{N}$?

Classically:

① n outputs compressible to $nH(X) + o(n)$ bits ["Source coding"]

$$H(X) := \sum -p_x \log p_x \quad \text{[Shannon Entropy]}$$

"Info/letter"

② n uses of $\mathcal{N}$ can communicate $nC - o(n)$ bits reliably [Channel coding]

$$C(x, \mathcal{N}) := \max_{p(x)} I(X : \mathcal{N}(X)) \quad \text{[Classical channel capacity]}$$

differentiating QM

$$I(X:Y) := H(Y) - H(Y|X) \quad \text{[Mutual information]}$$
$$= H(X) + H(Y) - H(X,Y) \quad \text{[From Bayes rule]}$$

Quantum:

① n outputs compressible to $nS(\rho) + o(n)$ qubits [Schumacher compression]

$$S(\rho) := -\operatorname{tr} \rho \log \rho \quad = H(\lambda(\rho))$$

② Many possible answers - depends on type of information to send & resources available.

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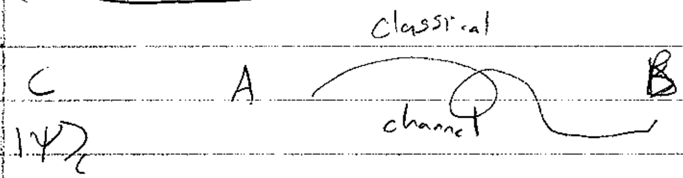
Teleportation



How to get $|\Psi_c\rangle$ to Bob?

Method 1: Bob prepares $|\mu\rangle_B = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$ at random

$$F_{avg} = \int d\varphi\, d\theta\, |\langle\mu|\Psi\rangle|^2 = 1/2 \qquad [\text{Preskill 1997 \#2.1}]$$

Method 2: Alice measures $|\Psi\rangle$ along $\hat{z}$-axis, tells Bob outcome, which Bob prepares.

interesting exercise: teleporting bits.

$$\rho_B = p_0 |0\rangle\langle 0| + p_1 |1\rangle\langle 1|,$$

$$\bar{p}_0 = \int d\varphi\, d\theta\, |\langle\psi|0\rangle|^2, \quad \bar{p}_1 = 1 - \bar{p}_0$$

$$F_{avg} = \int d\varphi\, d\varphi\, \langle\Psi|\bar{\rho}_B|\Psi\rangle$$

Best possible, actually →

$$= 2/3 \qquad [\text{Preskill 1997 \#2.2}]$$

Method 3: AB share $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. AC measure in Bell basis:

$$|\Psi_\pm\rangle\langle\Psi_\pm|_{AC},\ |\Phi_\pm\rangle\langle\Phi_\pm|_{AC}, \qquad |\Psi_\pm\rangle_{AC} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{AC}$$

$$(Z_A Z_C,\ X_A X_C) \qquad |\Phi_\pm\rangle_{AC} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{AC}.$$

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$$|\psi\rangle_C |\Phi^+\rangle_{AB} = (\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}(\underset{①}{\alpha|000\rangle} + \underset{②}{\alpha|011\rangle} + \underset{③}{\beta|100\rangle} + \underset{④}{\beta|111\rangle})$$

$$\begin{aligned} &= \tfrac{1}{2}\alpha(|\phi^+\rangle + |\phi^-\rangle)|0\rangle && ① \\ &+ \tfrac{1}{2}\alpha(|\psi^+\rangle + |\psi^-\rangle)|1\rangle && ② \\ &+ \tfrac{1}{2}\beta(|\psi^+\rangle - |\psi^-\rangle)|0\rangle && ③ \\ &+ \tfrac{1}{2}\beta(|\phi^+\rangle - |\phi^-\rangle)|1\rangle && ④ \end{aligned}$$

$$\begin{aligned} &= \tfrac{1}{2}|\phi^+\rangle(\alpha|0\rangle + \beta|1\rangle) \\ &+ \tfrac{1}{2}|\psi^+\rangle(\alpha|1\rangle + \beta|0\rangle) \\ &+ \tfrac{1}{2}|\psi^-\rangle(\alpha|1\rangle - \beta|0\rangle) \\ &+ \tfrac{1}{2}|\phi^-\rangle(\alpha|0\rangle - \beta|1\rangle) \end{aligned}$$

$$\begin{aligned} &= \tfrac{1}{2}|\phi^+\rangle_{CA}|\psi\rangle_B + \tfrac{1}{2}|\psi^+\rangle_{CA}X|\psi\rangle_B \\ &+ \tfrac{1}{2}|\psi^-\rangle_{CA}(iY)|\psi\rangle + \tfrac{1}{2}|\phi^+\rangle_{CA}Z|\psi\rangle_B \end{aligned}$$

Alice's outcome: 2 bits: B applies either $I, X, iY,$ or Z to his qubit $\rightarrow \boxed{|\psi\rangle_B!}$ $\boxed{F=1}$

Interesting exercise: pbit teleportation.

Werner state teleportation: $\rho_{AB} = (1-\lambda)|\psi^-\rangle\langle\psi^-| + \frac{\lambda}{4}I$

has $F > 2/3$ for $\lambda < 2/3$.

Note $\rho_C = \frac{1}{2}\mathbb{I}$ afterwards!

Generalizes to qudits: $X|a\rangle = |a \oplus 1\rangle$, $Z|a\rangle = \omega^a|a\rangle$.

(Measure XX, $ZZ \rightarrow d^2$ outcomes $\rightarrow 2\log d$ bits)

$\boxed{1 \text{ ebit} + 2 \text{ cbits} \rightarrow 1 \text{ qubit}}$ (1 edit + 2 cdits → 1 qudit)

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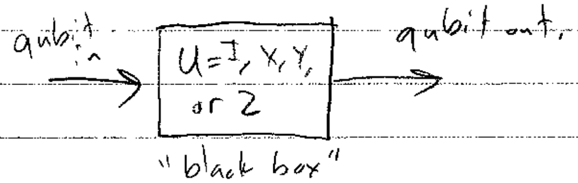
Superdense coding

Tomography viewpoint

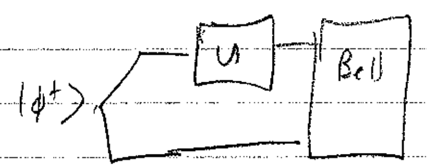


How many uses of box needed to find out what's inside?
("Quantum Mastermind")

2 uses suffice! (1) $|0\rangle$ in, $|0\rangle\langle 0|, |1\rangle\langle 1|$ out $\rightarrow$ $|0\rangle$ $U = I$ or Z; $|1\rangle$ $U = X$ or Y

(2) $|+\rangle$ in, $|+\rangle\langle +|, |-\rangle\langle -|$ out $\rightarrow$ $|+\rangle$ $U = I$ or X; $|-\rangle$ $U = Y$ or Z.

1 use suffices!



4 outcomes.

$$|\phi^-\rangle = Z \otimes I |\phi^+\rangle$$
$$|\psi^+\rangle = X \otimes I |\phi^+\rangle$$
$$|\psi^-\rangle = Y \otimes I |\phi^+\rangle$$
$$|\phi^+\rangle = I \otimes I |\phi^+\rangle.$$

Communication viewpoint:

A wants to send 2 bits to B in one qubit, AB share $|\phi^+\rangle$

Solution: A applies one of I, X, Y, Z to her half + sends to B.
B does Bell Meas $\rightarrow$ 4 outcomes = 2 bits!

Why "Superdense"? Holevo Bound: "No more than 1 bit/qubit" (roughly)

$\boxed{\text{The End}}$