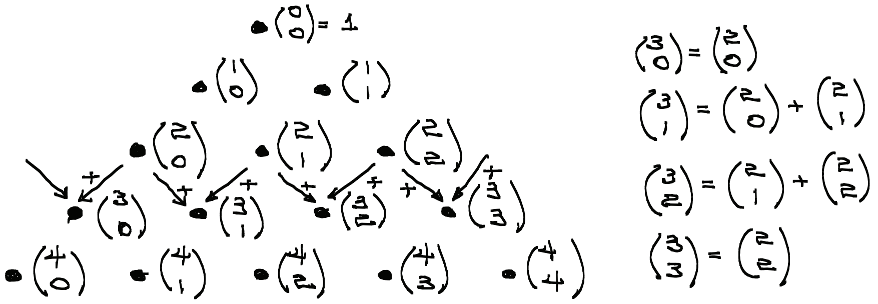
Phys 366 Mathematical Methods of Physics Fall 2016

Homework Assignment #10 Due Tuesday, November 29
(30 points) (at lecture)

10.1 (10 points) **Pascal's triangle.** You are probably familiar with Pascal's triangle (shown below) for generating the binomial coefficients.



Prove the fundamental result displayed in Pascal's triangle:

$$\binom{N}{n} = \binom{N-1}{n-1} + \binom{N-1}{n} .$$

10.2 (10 points) **Distinguishable particles, bosons, and fermions.**

(a) There are no fundamental distinguishable particles in Nature, but we can think about them anyway. Given N distinguishable particles and L (single-particle) states for them to occupy, one specifies an overall state by a list, $s_1, s_2, \ldots, s_N$, where s_j is the state occupied by particle j. What is the number $D_{N,L}$ of overall states available to N distinguishable particles that can occupy L states?

(b) Fermions are indistinguishable particles with half-integral spin, examples being electrons, protons, and neutrons; no two fermions are allowed to occupy the same (single-particle) state. Given a set of states, one specifies the overall state of the fermions by a list, $n_1, n_2, \ldots, n_L$, where $n_j = 0, 1$ is the number of particles in state j. What is the number $F_{N,L}$ of states available to N fermions that can occupy L states?

(c) Bosons are indistinguishable particles with integral spin, examples being various force carriers, i.e., photons, gravitons, and gluons. Bosons can occupy the same (single-particle) state. The overall state must be symmetric under interchange of the bosons, so it is impossible to say which particle occupies which state. Instead, one specifies an overall state by saying how many particles are in each single-particle state, i.e., by a list $n_1, n_2, \ldots, n_L$, where n_j is the number of bosons in state j. What is the number $B_{N,L}$ of overall states available to N bosons that can occupy L states? (Hint: This is the only hard one. Think about turning the list $n_1, \ldots, n_L$ into a bit sequence in which one places a 1 between the numbers n_j and replaces n_j by a sequence of n_j zeroes.)

(d) It should be clear that $D_{N,L} \geq B_{N,L} \geq F_{N,L}$. *Compare* the sizes of these three when $N \ll L$, and *interpret* what you find.

10.3 (10 points) Challenge problem.