Phys 366 Mathematical Methods of Physics Fall 2016

Homework Assignment #11 Due Thursday, December 8
(20 points) (at lecture)

11.1 (10 points) Consider the one-dimensional diffusion equation without a source on a line without boundaries:

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{\alpha^2}\frac{\partial f}{\partial t} = 0 \ .$$

A good way to solve this equation is to Fourier transform in space, so let's consider the following Fourier-transform pair:

$$f(x,t) = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, \tilde{f}(k,t) e^{ikx} \ , \qquad \tilde{f}(k,t) = \int_{-\infty}^{\infty} dx\, f(x,t) e^{-ikx} \ .$$

(a) *Find* the (ordinary) differential equation satisfied by $\tilde{f}(k,t)$, and and *solve* for the general solution with an initial value $\tilde{f}(k,0)$ at $t = 0$.

(b) Translate the general solution of part (a) back to the spatial domain to find the general solution for $f(x,t)$ in terms of the initial value $f(x,0)$. *Show* that the general solution is

$$f(x,t) = \int_{-\infty}^{\infty} dx'\, G(x-x',t) f(x',0) \ ,$$

where

$$G(x,t) = \frac{e^{-x^2/4\alpha^2 t}}{\sqrt{4\pi\alpha^2 t}}$$

is the temporal *Green function*. The Green function describes how an initial δ-spike in $f(x,0)$ diffuses as a Gaussian with a width that increases as $\alpha^2 t$.

11.2 (10 points) Challenge problem.