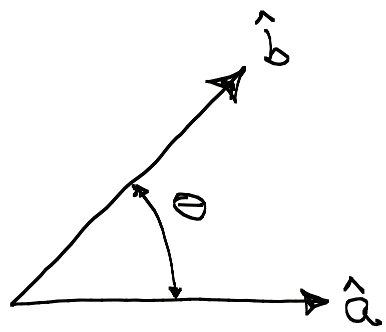
Phys 366 Mathematical Methods of Physics Fall 2016

Homework Assignment #2 Due Tuesday, September 6
(60 points) (at lecture)

2.1 (10 points) Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be *unit* vectors, separated by an angle θ, as shown in the drawing below. You will be asked to add to the drawing as part of the problem.



(a) *Give* the magnitude of the vector $\hat{\mathbf{a}}\times\hat{\mathbf{b}}$, and *describe* in words its direction.

(b) *Draw* and *label* the vector $\mathbf{A} = \hat{\mathbf{a}}\times(\hat{\mathbf{a}}\times\hat{\mathbf{b}})$ on your drawing, and *give* separately its exact magnitude.

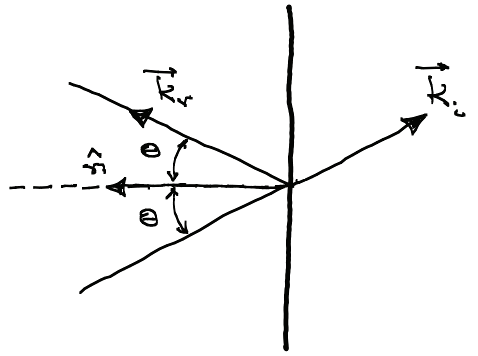
(c) *Draw* and *label* the vector $\mathbf{B} = \hat{\mathbf{b}}\times(\hat{\mathbf{a}}\times\hat{\mathbf{b}})$ on your drawing, and *give* separately its exact magnitude.

In drawing $\mathbf{A}$ and $\mathbf{B}$ in parts (b) and (c), you should indicate the directions as precisely as you can—use words if you need to—but you can be a bit careless about the magnitudes, because you must give the exact magnitudes in separate equations.

(d) *Give* the magnitude of the vector $\mathbf{C} = \mathbf{A}\times\mathbf{B}$, and *describe* in words its direction.

2.2 (10 points) Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be three vectors drawn from the origin to points A, B, and C. What is the distance from the origin to the plane defined by the points A, B, and C? What is the area of the triangle ABC, i.e., the triangle defined by the three points?

2.3 (10 points) Consider a planar surface with unit normal vector $\hat{\mathbf{n}}$. A light ray reflects from the surface, with the angle of incidence θ equal to the angle of reflection, as shown in the figure below. Let $\mathbf{k}_i$ and $\mathbf{k}_r$ be the incident and reflected wave vectors. As shown in the figure, the wave vectors point along the direction of propagation of the respective rays and have the same magnitude, $k = k_i = k_r = 2\pi/\lambda$, where λ is the wavelength.



(a) *Show* that

$$\mathbf{k}_r - \mathbf{k}_i = 2k\cos\theta\,\hat{\mathbf{n}}\ .$$

(b) *Show* that the reflected wave vector can be written as

$$\mathbf{k}_r = \mathbf{k}_i - 2\hat{\mathbf{n}}(\hat{\mathbf{n}}\cdot\mathbf{k}_i)\ .$$

This result completely characterizes how to construct the reflected wave vector from the incident wave vector and the surface normal.

(c) The result of part (b) implies immediately that

$$\begin{aligned}\hat{\mathbf{n}}\cdot\mathbf{k}_r &= -\hat{\mathbf{n}}\cdot\mathbf{k}_i\ ,\\ \hat{\mathbf{n}}\times\mathbf{k}_r &= \hat{\mathbf{n}}\times\mathbf{k}_i\ .\end{aligned}$$

Interpret these two conditions geometrically, and *explain* why they are sufficient to construct $\mathbf{k}_r$ from $\mathbf{k}_i$? Then, since they are sufficient, *derive* the result of part (b) from these two conditions.

2.4 (10 points) Let $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ be three position vectors.

(a) (10 points) *Give* a vector equation for the line $\mathbf{r}(s)$ that passes through the tips of $\mathbf{r}_1$ and $\mathbf{r}_2$. The line should be parametrized by the length s along the line, measured from the tip of $\mathbf{r}_1$ toward $\mathbf{r}_2$.

(b) (10 points) *Give* a vector expression for the distance d of the tip of $\mathbf{r}_3$ to the nearest point P on the line of part (a). What happens to your expression when the tip of $\mathbf{r}_3$ lies on the line?

(c) (10 points) *Give* the position vector $\mathbf{R}$ of point P.

2.5 (10 points) A plane passes through three points that lie on the Cartesian axes: $(x, y, z) = (1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

(a) *Find* a unit vector perpendicular to the plane.

(b) *Find* the distance from the point $(1, 1, 1)$ to the closest point in the plane and the coördinates of the closest point.

2.6 (10 points) Challenge problem