Fall 2016

Homework Assignment #5 (40 points)

Due Friday, September 30 (at lecture)

5.4 (10 points) Challenge problem. Consider a vector field  $\mathbf{A}$  defined in cylindrical coördinates by

$$\mathbf{A} = L\rho\,\hat{\mathbf{e}}_{\phi} + M\rho\sin\phi\,\hat{\mathbf{e}}_{z} \;,$$

where L and M are constants. The drawing shows a cylinder of radius a and height h, whose bottom lies in the x-y plane. Let the surface S consist of the side and the bottom of the cylinder (S is like a trash can; it does not include the top of the cylinder), and let  $d\mathbf{a}$  be the outward-directed area element on S, as indicated.



(a) Evaluate the surface integral

$$\int_{S} \nabla \times \mathbf{A} \cdot d\mathbf{a}$$

directly by doing the integral over the surface S.

(b) *Evaluate* the same surface integral by using Stokes's theorem to convert the surface integral into a line integral.

(c) Evaluate the same surface integral by using Stokes's theorem to convert the integral over S into a surface integral over some other surface than that specified in part (a).