**Phys 366 Mathematical Methods of Physics Fall 2016**

**Homework Assignment #7 Due Thursday, October 20**
**(30 points) (at lecture)**

7.3 (10 points) Challenge problem. Consider a two-dimensional quantum system that has Hamiltonian

$$H = \frac{1}{2}\hbar\omega\sigma_x \ ,$$

where

$$\sigma_x = |+,z\rangle\langle -,z| + |-,z\rangle\langle +,z| \quad \longleftrightarrow \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

is the Pauli $x$ operator. To say that $H$ is the Hamiltonian is to say that the state vector of the system changes in time according to the *Schrödinger equation*,

$$i\hbar\frac{d|\psi(t)\rangle}{dt} = H|\psi\rangle = \frac{1}{2}\hbar\omega\sigma_x|\psi(t)\rangle \ .$$

The state vector can be expanded in the $z$ basis as

$$|\psi(t)\rangle = c_+(t)|+,z\rangle + c_-(t)|-,z\rangle \ , \qquad c_\pm(t) = \langle \pm,z|\psi(t)\rangle \ .$$

(a) By projecting the Schrödinger equation onto the $z$ basis vectors, *derive* the evolution equations for $c_+(t)$ and $c_-(t)$. *Solve* the equations assuming that the system begins in the state $|+,z\rangle$.

The Pauli $x$ operator can be diagonalized in its eigenbasis as

$$\sigma_x = |+,x\rangle\langle +,x| - |-,x\rangle\langle -,x| \ ,$$

where the eigenvectors are given by

$$|\pm,x\rangle = \frac{1}{\sqrt{2}}(|+,z\rangle \pm |-,z\rangle) \ .$$

The state vector can be expanded in the $x$ basis as

$$|\psi(t)\rangle = d_+(t)|+,x\rangle + d_-(t)|-,x\rangle \ , \qquad d_\pm(t) = \langle \pm,x|\psi(t)\rangle \ .$$

(b) By projecting the Schrödinger equation onto the $x$ basis vectors, *derive* the evolution equations for $d_+(t)$ and $d_-(t)$. *Solve* the equations assuming that the system begins in the state $|+,z\rangle$.

(c) Use your solution to part (b) to *rederive* the solution to part (a).

(d) What is the probability for the two results, $+$ and $-$, in a measurement of $\sigma_z$ at time $t$? In a measurement of $\sigma_x$ at time $t$?