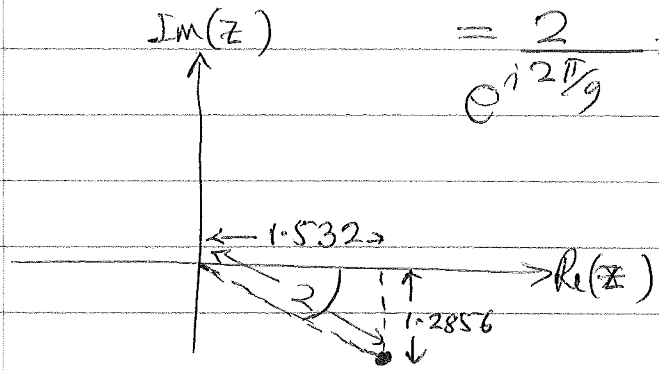
~~-0-~~

1.1

~~1.~~ Prob-16, Sec 2.5

$$\frac{1}{0.5(\cos 40^\circ + i\sin 40^\circ)} = \frac{2}{\cos\left(\frac{40\times\pi}{180}\right) + i\sin\left(\frac{40\pi}{180}\right)}$$



$$= \frac{2}{e^{i2\pi/9}} = 2e^{-i2\pi/9} \quad (re^{i\theta}\text{ form})$$

$$= 2\cos 2\pi/9 - i2\sin 2\pi/9$$

$$= 1.532 - i1.2856$$

$$(x+iy \text{ form})$$

Prob-28, Sec 2.5

$$\left|\frac{z}{\bar{z}}\right| = \frac{|z|}{|\bar{z}|} = \frac{|re^{i\theta}|}{|re^{-i\theta}|} = \frac{r}{r} = 1$$

Note: The absolute value of a product or ratio of two complex #s is the product or ratio of the absolute values of the two #s. However, the absolute value of the sum or difference of two complex #s is not the sum or difference of the two absolute values!!

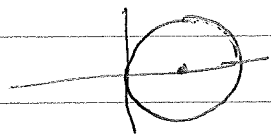
Prob-33, Sec 2.5

$$\left|\frac{25}{3+4i}\right| = \frac{|25|}{|3+4i|} = \frac{25}{\sqrt{3^2+4^2}} = \frac{25}{5}$$

$$= 5$$

Prob 58, Sec 2.5

$$|z-1| = 1 \Rightarrow |x-1+iy| = 1 \Rightarrow \sqrt{(x-1)^2+y^2} = 1$$



or $(x-1)^2+y^2=1$ ← eq'n of a circle centered at the point $(1,0)$ and with radius 1

Prob 62, Sec 2.5

$$|z+1|+|z-1| = 8 \Rightarrow |x+1+iy| + |x-1+iy| = 8$$

ie. $\sqrt{(x+1)^2+y^2} + \sqrt{(x-1)^2+y^2} = 8$

or, $(x+1)^2+\cancel{y^2} = \left(8-\sqrt{(x-1)^2+y^2}\right)^2$

$$= 64 - 16\sqrt{(x-1)^2+y^2} + (x-1)^2 + \cancel{y^2}$$

$\Rightarrow \quad 4x = 64 - 16\sqrt{(x-1)^2+y^2}$

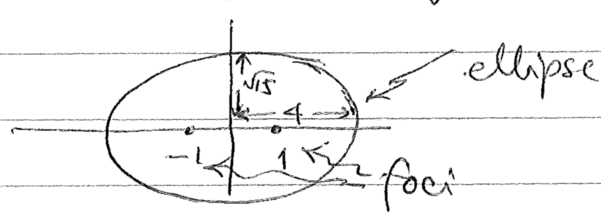
ie, $16 - x = 4\sqrt{(x-1)^2+y^2}$

Square $\Rightarrow \quad 256 - 32x + x^2 = 16(x-1)^2 + 16y^2$

$\Rightarrow \quad 16y^2 + 15x^2 = 240$

$\Rightarrow \quad \dfrac{x^2}{16} + \dfrac{y^2}{15} = 1$

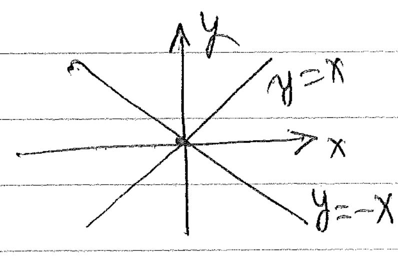
Eq'n of an ellipse of semimajor axis 4 and semiminor axis length $\sqrt{15}$ along the x-y axes



Prob 64, Sec 2.5

$$z^2 = -\bar{z}^2$$

ie, $x^2-y^2+2ixy = -(x^2-y^2-2ixy)$



$\Rightarrow \quad x^2 = y^2 \Rightarrow y = \pm x$: 45°, 135° orientation lines thro' the origin

Perhaps easier: $r^2 e^{2i\theta} = z^2 = -\bar{z}^2 = -r^2 e^{-2i\theta}$

$$\Rightarrow e^{4i\theta} = -1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \text{ or } \frac{7\pi}{4}$$

$45°, 135°, 225°, 315°$