9.2. This is straightforward. All we have to do is to manipulate the formula for the Fourier transform:

$$\begin{aligned}
\tilde{f}(k) &= \int_{-\infty}^{\infty} dx\, f(x) e^{-ikx} \\
&= \int_{-L/2}^{L/2} dx\, f(x) e^{-ikx} \qquad \begin{pmatrix} f(x) \text{ is only zero on} \\ \text{the interval } [-L/2, L/2]. \end{pmatrix} \\
&= \int_{-L/2}^{L/2} dx\, f_L(x) e^{-ikx} \qquad \begin{pmatrix} f(x) = f_L(x) \text{ on} \\ \text{the interval } [-L/2, L/2]. \end{pmatrix} \\
&= \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} c_n \underbrace{\int_{-L/2}^{L/2} dx\, e^{i(k_n - k)x}}_{= \dfrac{\sin[(k_n - k)L/2]}{(k_n - k)/2}} \qquad \begin{pmatrix} \text{Plug in the Fourier} \\ \text{series for } f_L(x). \end{pmatrix} \\
&= \sum_{n=-\infty}^{\infty} \sqrt{L} c_n \frac{\sin[(k - k_n)L/2]}{(k - k_n)L/2} \;,
\end{aligned}$$

We have

$$\tilde{f}(k_n) = \frac{1}{\sqrt{L}} \sum_{m=-\infty}^{\infty} c_m \tilde{\Theta}\big((k_n - k_m)\big) = \sum_{m=-\infty}^{\infty} \sqrt{L} c_m \underbrace{\frac{\sin(n-m)\pi}{(n-m)\pi}}_{= \delta_{nm}} = \sqrt{L} c_n$$

The sinc function in the formula above is there to restrict $f(x)$ to the interval $[-L/2, L/2]$. It occurs wherever such a restriction is needed. Indeed, a more compact way to write this is to introduce the function

$$\Theta(x) = \begin{cases} 1\,, & \text{for } |x| < L/2, \\ 0\,, & \text{for } |x| > L/2, \end{cases}$$

whose Fourier transform is the sinc function,

$$\tilde{\Theta}(k) = \int_{-\infty}^{\infty} dx\, \Theta(x) e^{-ikx} = \int_{-L/2}^{L/2} dx\, e^{-ikx} = L \frac{\sin(kL/2)}{kL/2} \;.$$

Our formulas can then be written as

$$f(x) = f_L(x)\Theta(x)\;, \qquad \tilde{f}(k) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} c_n \tilde{\Theta}\big((k - k_n)\big) \;.$$

To summarize: To restrict a periodic function to one of its intervals, one fuzzes out the Fourier-series coefficients using the sinc function; the sinc function spreads out the superposition of harmonic functions in just the right way to get constructive interference in one of the intervals and complete destructive interference in all the others. Kind of remarkable.

What you should remember: The Fourier transform of a function that is restricted to a domain of finite size can be reconstructed, using the sinc function, from the values of the Fourier transform at a series of uniformly spaced points.