Phys 366 Mathematical Methods of Physics Fall 2016

Quiz 1 2016 September 13
(100 points)

Problem 1 (45 points)

(a) (10 points) Using Euler's formula, $e^{ix} = \cos x + i \sin x$, *show* that $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ and $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$. Note that I use the standard, but sometimes confusing notation that $\cos^2 x = (\cos x)^2$ and similarly for the sine function.

(b) (25 points) Using the formula for the sum of a geometric series,

$$\sum_{n=0}^{N} x^n = \frac{1 - x^{N+1}}{1 - x} ,$$

find the finite sum

$$S_N = \sum_{n=0}^{N} R^{2n} \sin^2 n\theta ,$$

where R and θ are real numbers.

(c) (10 points) *Find* $\lim_{N\to\infty} S_N$. For what values of R does this limit exist?

Problem 2 (55 points)

Consider the plane $\mathcal{M}$ defined by the tips of the three vectors $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}}$, $\mathbf{B} = \hat{\mathbf{y}}$, and $\mathbf{C} = \hat{\mathbf{z}}$, all of which have their tails at the origin.

(a) (10 points) *Show* that the unit vector $\hat{\mathbf{m}}$ that is normal to $\mathcal{M}$ (and has nonnegative Cartesian components) is

$$\hat{\mathbf{m}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{y}} + \hat{\mathbf{z}}) .$$

(b) (10 points) The plane $\mathcal{M}$ is defined by the equation $\hat{\mathbf{m}} \cdot \mathbf{r} = H$, where H is the distance from the origin to the closest point on $\mathcal{M}$. *Find* the value of H.

Now let $\mathcal{L}$ be the line through the origin defined by $\mathbf{r}(s) = s\hat{\mathbf{n}}$, where

$$\hat{\mathbf{n}} = \frac{3}{5}\hat{\mathbf{y}} + \frac{4}{5}\hat{\mathbf{z}}$$

is a unit vector whose tail is at the origin and s measures length along the line.

(c) (15 points) *Find* the value of s at which $\mathcal{L}$ intersects $\mathcal{M}$ and, hence, the vector $\mathbf{D} = s\hat{\mathbf{n}}$ on $\mathcal{L}$ whose tip lies in $\mathcal{M}$.

(d) (20 points) Consider an arbitrary point $\mathbf{r}(s)$ on $\mathcal{L}$. *Find* the distance d_1 from $\mathbf{r}(s)$ to the nearest point on $\mathcal{M}$ and the distance d_2 of this nearest point from the intersection of $\mathcal{L}$ with $\mathcal{M}$.