Phys 366 **Mathematical Methods of Physics** **Fall 2016**

Quiz 3 **2016 October 27**
(100 points)

Problem 1 (100 points) Let $|e_k\rangle$, $k = 1, 2, 3$, be an orthonormal basis for a three-dimensional complex vector space. Consider the operator

$$Y = |e_3\rangle\langle e_1| + |e_1\rangle\langle e_2| + |e_2\rangle\langle e_3| \quad \longleftrightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} .$$

This operator is called the *lowering operator*, because it takes $|e_3\rangle$ to $|e_2\rangle$, $|e_2\rangle$ to $|e_1\rangle$, and $|e_1\rangle$ to $|e_3\rangle$.

(a) (20 points) *Write* the adjoint, $Y^\dagger$, in bra-ket notation, and *give* its matrix representation in the $|e_k\rangle$ basis. *Show* that Y is a unitary operator (i.e., show that $YY^\dagger = I$).

(b) (60 points) *Find* the eigenvectors and eigenvalues of Y. Unitary operators have a complete, orthonormal set of eigenvectors, and their eigenvalues are phases. Label the eigenvectors of Y by the corresponding eigenvalue.

(c) (20 points) *Write* Y^2 in bra-ket notation, and *give* its matrix representation. How is Y^2 related to $Y^\dagger$? *Find* the eigenvectors and eigenvalues of Y^2. [Hint: This last is very easy if you do it the right way; i.e., you don't have to solve another eigenvalue problem. But be sure you have done (a) and (b) correctly before you mess with this part.]