Phys 366 Mathematical Methods of Physics Fall 2016

Quiz 5 2015 November 22
(100 points)

Problem 1 (100 points)

(a) (25 points) *Show* that the temporal Fourier transform of the function

$$s(t) = H(t)e^{-\beta t} = \begin{cases} 0\,, & t < 0, \\ e^{-\beta t}\,, & t \geq 0, \end{cases}$$

where β is a positive constant, is

$$\tilde{s}(\omega) = \frac{1}{\beta - i\omega}$$

The velocity v of a particle of mass m subjected to a viscous damping force $-m\beta v$ (as in air), where β is a constant, and to an external force $f(t)$ satisfies the ordinary differential equation,

$$\dot{v} + \beta v = \frac{f(t)}{m}\,. \tag{1}$$

(b) (20 points) *Show* that the temporal Fourier transform of $v(t)$ satisfies

$$\tilde{v}(\omega) = \frac{1}{m}\frac{\tilde{f}(\omega)}{\beta - i\omega}\,.$$

(c) (40 points) By inverting the Fourier transform to find $v(t)$, *show* that the solution of Eq. (1) has the form

$$v(t) = \int_{-\infty}^{t} g(t-t')\frac{f(t')}{m}\,,$$

and *find* the function $g(t-t')$, which is called the causal Green function.

(d) (15 points) *Show* directly that your solution in part (c) satisfies Eq. (1).

Temporal Fourier transform pair

$$s(t) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\tilde{s}(\omega)e^{-i\omega t} \qquad\qquad \tilde{s}(\omega) = \int_{-\infty}^{\infty} dt\, s(t)e^{i\omega t}$$