Phys 503 Classical Mechanics I **Fall 2013**

Midterm Exam #1 **Due Friday, October 4**
(20% of course grade) **(at noon in TA's mailbox)**

This is an examination for Phys 503. Your score will determine 20% of your grade for the course.

This is a take-home, open-book exam. You may use the textbook, your own notes, your own homework assignments, all class handouts, including solution sets for homework assignments, but do not consult books or any other material, including online material, other than our primary textbook.

You may use as much time as you wish to complete the exam, as long as it is turned in by the deadline. Your completed exam must be solely your own work; do not consult anyone else in doing the exam.

Do your work on sheets of paper separate from the exam. The solution you hand in for a problem should be neat and legible and in a logical order. It should represent your best effort at solving the problem and should not include false starts and detours that led nowhere.

Staple your solution(s) to this page, and label this page with your *printed* name. Then *sign and date the pledge below,* and turn in the exam as instructed. You may keep the exam question(s).

I have obeyed the rules in taking this exam. In particular, I have not consulted anyone about the exam, and the completed exam is solely my own work.

____ ____________________

Signature Date

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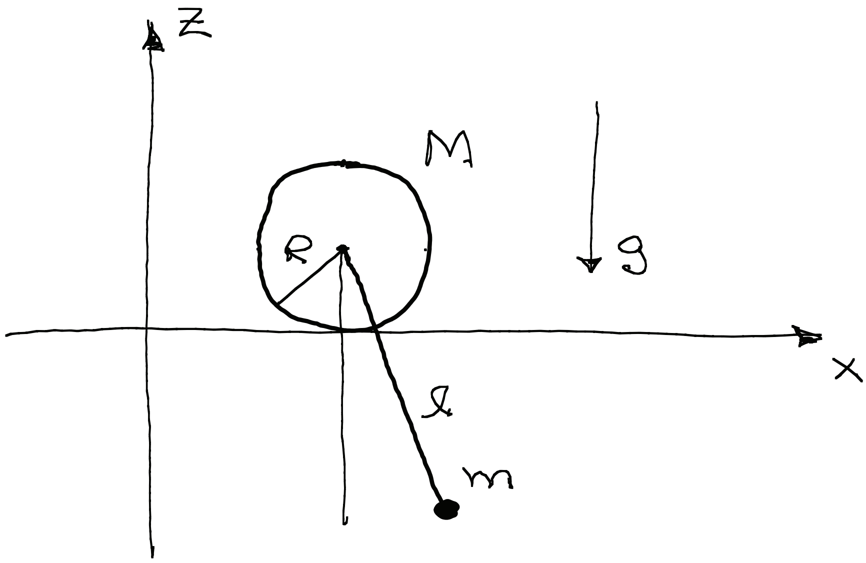
Midterm Exam #1 **(100 points)**

Problem 1 (25 points) A hollow cylinder of mass M and radius R rolls without slipping on a table. The end of the cylinder hangs off the edge of the table and is constructed in such a way that a pendulum is suspended from the axis of the cylinder. The pendulum rod, assumed rigid and massless, has length l, and the pendulum bob has mass m. Assume that the pendulum can rotate without friction in its support.



(a) (5 points) Define carefully a set of *independent* generalized coördinates for this system.

(b) (10 points) Give a Lagrangian L in terms of the generalized coördinates you defined in (a).

(c) (10 points) Give two conserved quantities for this system.

Problem 2 (40 points) A particle of mass m in a Kepler central potential $V(r) = -k/r$ has orbits described by

$$\frac{1}{r} = \frac{mk}{l^2}(1 + e\cos\theta),$$

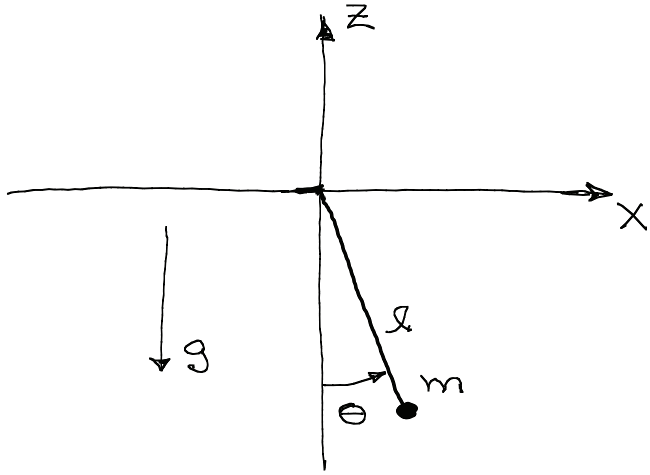
where

$$e = \sqrt{1 + \frac{2l^2E}{mk^2}}.$$

(a) (20 points) Suppose the particle is initially in a parabolic orbit. An impulse is applied at periastron to place the particle in a circular orbit. Give the energy and angular momentum of the circular orbit in terms of the energy and angular momentum of the initial parabolic orbit, and characterize *completely* the required impulse. Draw an effective-potential diagram that shows the transition between the two orbits.

(b) (20 points) Suppose the particle is initially in an arbitrary elliptical orbit. An impulse is applied at $\theta = \pi/2$ to place the particle in a circular orbit. Give the energy and angular momentum of the circular orbit in terms of the energy and angular momentum of the initial orbit, and characterize *completely* the required impulse. Draw an effective-potential diagram that shows the transition between the two orbits.

Problem 3 (35 points) A particle of mass m is suspended from a massless string of length l and is constrained to move in the x-z plane.



(a) (10 points) Write the Lagrangian that treats the tension in the string using a Lagrange multiplier, derive from it the equations of motion of m, and identify the string tension $\mathcal{T}$ in the equations

(b) (25 points) Suppose the mass, initially at rest at its equilibrium position, is given an impulse S in the positive x direction. Determine the range of impulses for which the string crumples and, for impulses in that range, the angle θ_c at which the string crumples.