Phys 503 Classical Mechanics I **Fall 2013**

Midterm Exam #2 **Due Tuesday, November 12**
(20% of course grade) **(at lecture)**

This is an examination for Phys 503. Your score will determine 20% of your grade for the course.

This is a take-home, open-book exam. You may use the textbook, your own notes, your own homework assignments, all class handouts, including solution sets for homework assignments, but do not consult books or any other material, including online material, other than our primary textbook.

You may use as much time as you wish to complete the exam, as long as it is turned in by the deadline. Your completed exam must be solely your own work; do not consult anyone else in doing the exam.

Do your work on sheets of paper separate from the exam. The solution you hand in for a problem should be neat and legible and in a logical order. It should represent your best effort at solving the problem and should not include false starts and detours that led nowhere.

Staple your solution(s) to this page, and label this page with your *printed* name. Then *sign and date the pledge below,* and turn in the exam as instructed. You may keep the exam question(s).

I have obeyed the rules in taking this exam. In particular, I have not consulted anyone about the exam, and the completed exam is solely my own work.

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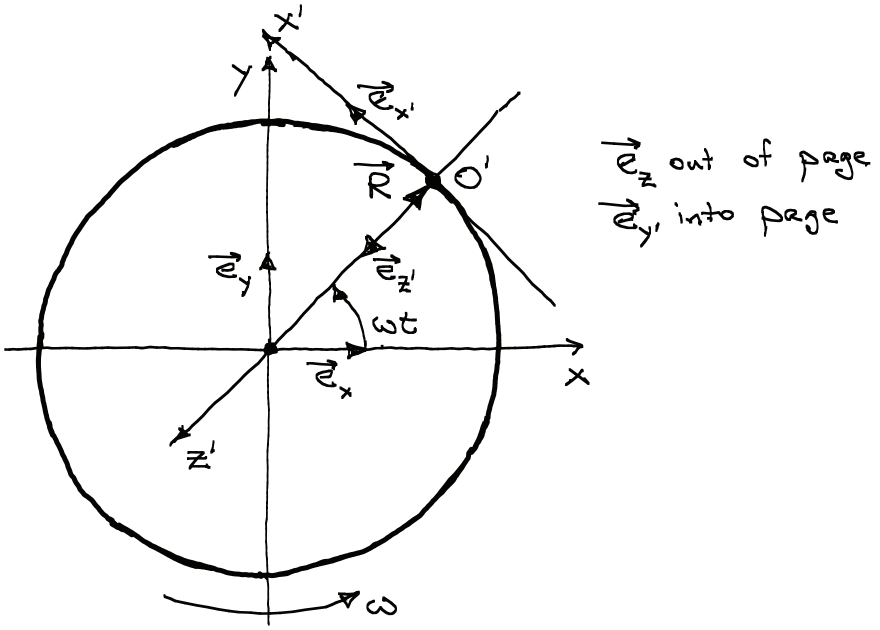
Signature Date

Phys 503 Classical Mechanics I **Fall 2013**
Midterm Exam #2 **(100 points)**

Problem 1 (30 points) The kind of space colony proposed by Gerard O'Neill in the 70's was a large cylinder of radius $R = 10\,\mathrm{km}$ and length $l = 100\,\mathrm{km}$, set spinning with angular velocity ω about its axis. We consider an inertial frame with origin O on the axis of rotation and z axis along the axis of rotation. A space colonist sets up a noninertial frame in his vicinity: the origin O' is at some point on the cylinder (the colonist's "ground"), the z' axis is in the direction toward the center of the cylinder, i.e., along the local "vertical," the x' axis is tangent to the cylinder and in the direction of the rotational velocity, and the y' axis is along the length of the cylinder.

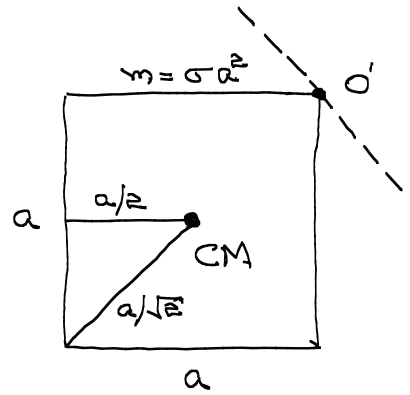


(a) (5 points) Write the angular velocity vector $\boldsymbol{\omega}$ and the vector $\mathbf{R}$ and its first two (inertial-frame) time derivatives in terms of basis vectors in the inertial frame and in the space colonist's frame.

(b) (10 points) Find the equations of motion of a particle in the space colonist's frame; i.e., find the equations of motion for x', y', and z'. Assume the particle is not subject to any real forces.

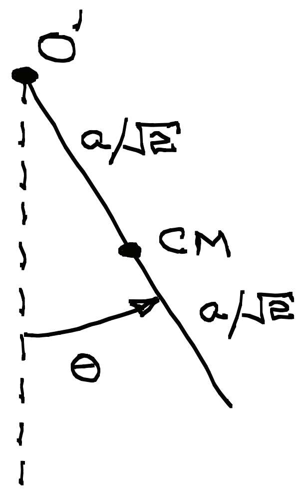
(c) (15 points) Suppose a space colonist drops a particle from rest from the height h of his head. Assume that $h \ll R$ and, hence, that the time before the particle hits the "ground" is small compared to a rotation period, i.e., $\omega t \ll 1$ for all times of interest. With these assumptions, find the leading-order time-dependent behavior of $x'(t)$ and $z'(t)$. (The exact trajectory for later times describes the motion of a particle that passes through a hole in the cylinder, continuing on a straight line in inertial space, but executing an apparently complicated motion in the space colonist's coördinates.) If the angular velocity is adjusted so that the inhabitants of the space colony experience a local "acceleration of gravity" equal to Earth's, would the deviation of the particle from such a vertical drop be noticeable to the colonist?

Problem 2 (35 points) Consider a rigid body in the shape of a square of negligible thickness. The square has side length a, uniform surface mass density $\sigma = m/a^2$, and total mass m.



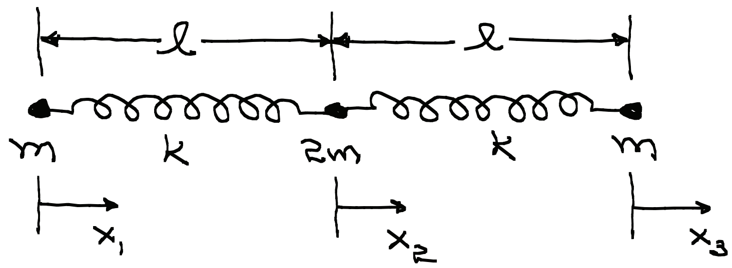
(a) (13 points) Suppose we are interested in rotations of the square about its center of mass (the point CM in the above drawing) and thus in the inertia tensor $\mathbf{I}_{\rm CM}$ about the center of mass. Draw and/or describe clearly principal-axis directions for $\mathbf{I}_{\rm CM}$, and give the corresponding principal moments of inertia. Are the principal-axis directions unique? If not, describe completely the freedom that is available in choosing the principal axes.

(b) (12 points) Suppose we are interested in rotations of the square about the point O' in the above drawing and thus in the inertia tensor $\mathbf{I}_{O'}$ about O'. Draw and/or describe clearly principal-axis directions for $\mathbf{I}_{O'}$, and give the corresponding principal moments of inertia. Are the principal-axis directions unique? If not, describe completely the freedom that is available in choosing the principal axes.



(c) (10 points) Suppose the square is suspended from point O' in such a way that it can only rotate about the dotted axis shown in the top drawing (the drawing immediately above shows a side view of the rotating square). Find the kinetic energy T and potential energy V in terms of the angular coördinate θ shown above. What is the frequency ω of small oscillations about $\theta = 0$?

Problem 3 (35 points) A linear triatomic molecule can be modeled as three masses connected by two springs (see drawing). Assume that both springs have unstretched length l and spring constant k and that the outer atoms have mass m and the middle atom has mass $2m$.



(a) (13 points) Using the coördinates x_1, x_2, and x_3, which give displacements away from mechanical equilibrium, find the kinetic-energy matrix $\mathbf{T}$ and the potential-energy matrix $\mathbf{V}$.

(b) (10 points) Determine the oscillation frequencies ω_k of the normal modes.

(c) (12 points) Give eigenvectors $\mathbf{a}_k$ for the normal modes. You don't need to normalize the eigenvectors. [You might find it convenient to do parts (b) and (c) at the same time.]