

Homework Assignment #3
(60 points)

Due Thursday, October 17
(at lecture)

3.1 (10 points) Goldstein 4.18 (translated)

(a) Find the vector equation that describes the (active) reflection of a vector \mathbf{x} through a plane whose unit normal is \mathbf{n} .

(b) Show that the matrix of this transformation has the elements

$$A_{jk} = \delta_{jk} - 2n_j n_k,$$

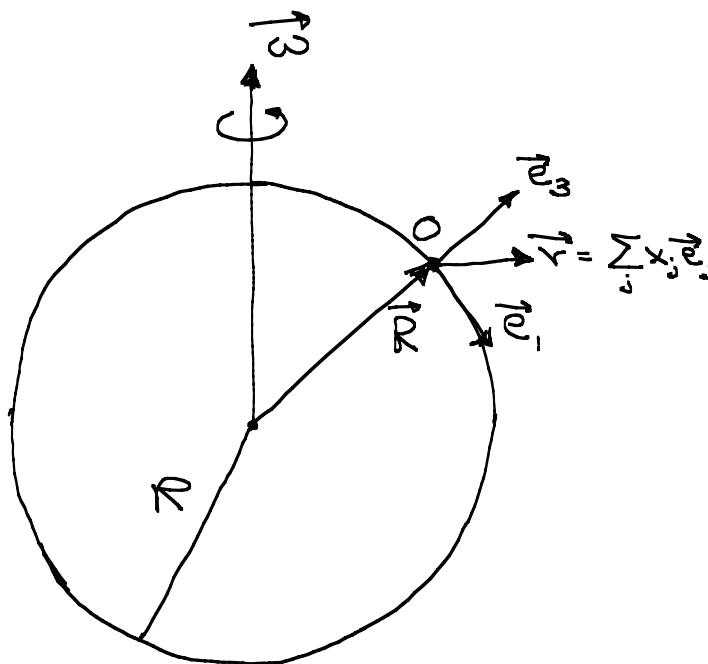
and verify explicitly that \mathbf{A} is an improper orthogonal matrix.

3.2 (10 points) Goldstein 4.7

3.3 (10 points) Goldstein 4.10 (a)–(d) plus

(e) Show that $(e^{i\mathbf{B}})^\dagger = e^{-i\mathbf{B}^\dagger}$ and that if \mathbf{B} is Hermitian, then $e^{i\mathbf{B}}$ is unitary.

3.4 (10 points) A rotating frame, with basis vectors \mathbf{e}_j , has its origin O attached to the surface of the earth, as shown in the drawing below (we are omitting the primes that we usually use to distinguish a rotating frame).



The earth rotates with angular velocity $\boldsymbol{\omega}$. The vector from the center of the earth to O is denoted by \mathbf{R} . The position of a particle relative to O is specified by a vector

$$\mathbf{r} = x_j \mathbf{e}_j.$$

The particle has mass m and potential energy $V = V(x_1, x_2, x_3)$. We wish to describe its motion in terms of the non-inertial coordinates x_j (you should recognize that we are talking about a description in terms of centrifugal and Coriolis forces). The velocity of the particle relative to inertial space is

$$\mathbf{w} = \left(\frac{d(\mathbf{R} + \mathbf{r})}{dt} \right)_{\text{space}} .$$

One objective of this problem is to gain experience in using the summation convention, the Kronecker delta, and the antisymmetric symbol. You must use these mathematical tools to do the problem and to express your answers.

- (a) What are the components X_j of \mathbf{R} in the rotating basis \mathbf{e}_j ?
 - (b) Give the components w_j of \mathbf{w} in the rotating basis \mathbf{e}_j .
 - (c) Give the Lagrangian of the particle in terms of the non-inertial coordinates x_j and the corresponding generalized velocities \dot{x}_j .
 - (d) (10 points) Derive the particle's equation of motion from the Lagrangian.
- 3.5 (10 points) Goldstein 4.22
- 3.6 (10 points) Challenge problem