

Homework Assignment #6
(70 points)Due Thursday, November 21
(at lecture)

6.7 (10 points) Challenge problem. Consider a frame that rotates with constant angular velocity. Let the axes in the rotating frame be defined by rotating basis vectors \mathbf{e}_j , in terms of which the angular velocity is $\boldsymbol{\omega} = \omega_j \mathbf{e}_j$. A particle of mass m , with position vector $\mathbf{x} = x_j \mathbf{e}_j$, moves in an arbitrary potential $V(x_1, x_2, x_3)$, which is time independent in the rotating frame.

(a) Find the particle's Lagrangian $L = T - V$ in terms of the rotating coordinates x_j .

(b) Derive the canonical momenta p_j , and describe in words what physical quantity they represent.

(c) Derive the particle's Hamiltonian H in terms of the canonical coordinates x_j and the canonical momenta p_j .

(d) Suppose $V = 0$. Is the Hamiltonian equal to the particle's total energy $E = T$? Is the Hamiltonian conserved? Is the energy E conserved? Once you've decided on answers to these questions, explain in words how they are consistent with one another.

(e) Redo part (d) when $V \neq 0$.