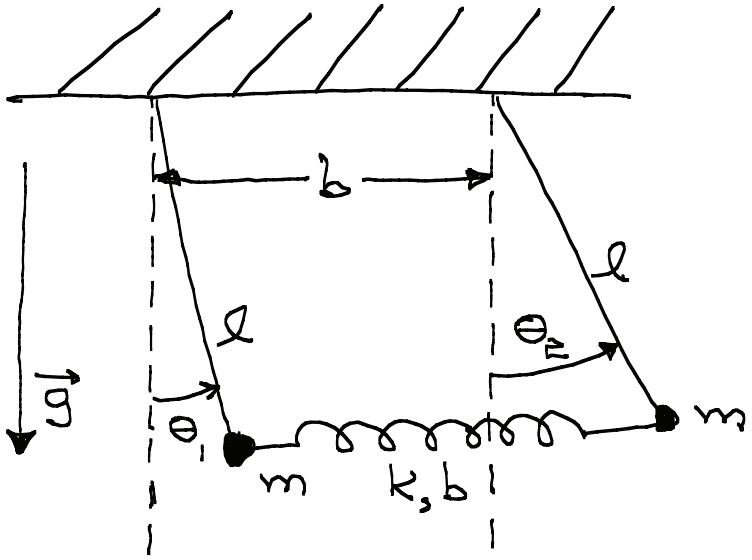
Phys 503 Classical Mechanics I **Fall 2013**

Homework Assignment #7 **Due Tuesday, December 3**
(60 points) **(at lecture)**

7.5 (10 points) Challenge problem (a). Consider the coupled pendulum shown below. Two equal masses m hang from strings of length l. The masses are coupled by a spring that has spring constant k and whose unstretched length b is equal to the distance between the strings' supports.



Throughout this problem you are to use the approximation of small oscillations.

(a) Give the Lagrangian L in terms of the generalized coördinates θ_1 and θ_2.

(b) Give the frequencies ω_1 and ω_2 of the normal modes and the (normalized) normal coördinates Q_1 and Q_2, i.e., the coördinates such that the the Lagrangian has the form

$$L = \sum_{j=1}^{2} \frac{1}{2}(\dot{Q}_j^2 - \omega_j^2 Q_j^2).$$

Express the relation between the normal coördinates and the original coördinates as $Q_j = A_{jk}\theta_k$, where $\mathbf{A}$ is a matrix. (You should be able to guess the normal coördinates and thereby avoid the formal procedure of diagonalizing matrices.)

(c) Let p_j denote the canonical momentum conjugate to q_j, and let P_j denote the canonical momentum conjugate to Q_j. Find the relation between the two sets of momenta, and express it in terms of the matrix $\mathbf{A}$ found in part (b).

(d) Find a generating function $F_2(\theta, P)$ for the canonical transformation from the original coördinates and momenta to the normal coördinates and momenta.