Phys 503 Classical Mechanics I **Fall 2013**

Homework Assignment #7 **Due Tuesday, December 3**
(60 points) **(at lecture)**

7.6 (10 points) Challenge problem (b). Suppose a particle moving in one dimension is described by the Hamiltonian $H = qp$.

(a) What are the Hamilton equations of motion for q and p? Give the solutions for $q(t)$ and $p(t)$ in terms of the initial values q_0 and p_0.

(b) Using any method that works, show that the coördinate transformation

$$X = \frac{1}{\sqrt{2}}(q - p), \qquad \Pi = \frac{1}{\sqrt{2}}(q + p),$$

is a canonical transformation. Find the transformed Hamiltonian, derive from the transformed Hamiltonian the Hamilton equations of motion for X and Π, and give the solutions for $X(t)$ and $\Pi(t)$ in terms of the initial values X_0 and Π_0. [Hint: Perhaps you should check that these solutions are consistent with those in part (a).]

(c) Draw a 2-dimensional phase space with q labeling the horizontal axis and p labeling the vertical axis. On this phase space draw the axes corresponding to X and Π. In each quadrant draw a typical trajectory, indicating the direction of motion along the trajectory with an arrow.

(d) Using any method that works, find a function $f(P)$ such that the coördinate transformation

$$q = e^{Q}, \qquad p = f(P)e^{-Q},$$

is a canonical transformation. Give a generating function for the transformation, and find the transformed Hamiltonian.