Phys 521 Quantum Mechanics I **Fall 2011**

Exam #1 **Due Tuesday, September 20**

(16% of course grade) **(at noon in TA's mailbox)**

This is an examination for Phys 521. Your score will determine 16% of your grade for the course.

This is a take-home, open-book exam. You may use the textbook, your own notes, your own homework assignments, all class handouts, including solution sets for homework assignments, but do not consult books other than our primary textbook. You may use as much time as you want to complete the exam, as long as it is turned in by the deadline. Your completed exam must be solely your own work; do not consult anyone else in doing the exam.

Do your work on sheets of paper separate from the exam. The solution you hand in for a problem should be neat and legible and in a logical order. It should represent your best effort at solving the problem and should not include false starts and detours that led nowhere.

Staple your solution(s) to this page, and label this page with your *printed* name. Then *sign and date the pledge below,* and turn in the exam as instructed. You may keep the exam question(s).

I have obeyed the rules in taking this exam. In particular, I have not consulted anyone about the exam, and the completed exam is solely my own work.

____ ____________________

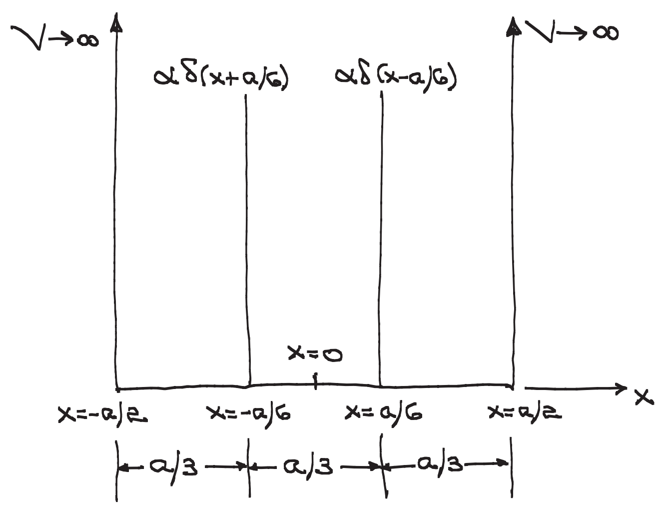
Signature Date

Phys 521 **Quantum Mechanics I** **Fall 2011**
Exam #1 **(100 points)**

Problem 1 (65 points) A particle of mass m moves in the potential

$$V(x) = \begin{cases} \alpha\delta(x + a/6) + \alpha\delta(x - a/6)\ , & -a/2 < x < a/2, \\ \infty\ , & |x| > a/2. \end{cases}$$

The potential is an infinite square well of width a, which contains two δ-function barriers of equal strength α, one at $x = a/6$ and the other at $x = -a/6$. The δ barriers divide the well into three regions, each of width $a/3$. The potential is depicted below.



(a) (20 points) When $\alpha = 0$, the bound states are those of an infinitely deep well. When $\alpha \to \infty$, the δ barriers divide the infinite well into three isolated wells, each of width $a/3$; as a result, the ground state becomes three-fold degenerate. The lowest three bound states for $\alpha = 0$ transform to three degenerate and orthogonal ground states in the limit $\alpha \to \infty$. For each of the lowest three bound states, *sketch* the wave function as it changes from $\alpha = 0$ to a typical value of α between 0 and ∞ and then to $\alpha \to \infty$; *justify* your sketches in words. You should do this now, but return to this part to update your sketches and justifications in light of what you find in the remainder of the problem.

The remainder of the problem focuses on the ground-state wave function and energy. The ground-state wave function is even, so we write it as

$$\varphi(x) = \begin{cases} \cos kx\ , & |x| < a/6, \\ B\cos(k|x| - \theta)\ , & a/6 < |x| < a/2, \\ 0\ , & |x| > a/2, \end{cases}$$

where $k = \sqrt{2mE}/\hbar$. We aren't going to worry about normalizing this wave function; instead, we get rid of one constant by letting the wave function in the middle region be $\cos kx$.

(b) (15 points) Use the boundary conditions to *determine* how k and θ are related and to *show* that

$$2B \sin \frac{ka}{6} = 1 \; .$$

(c) (15 points) Use the boundary conditions to *show* that

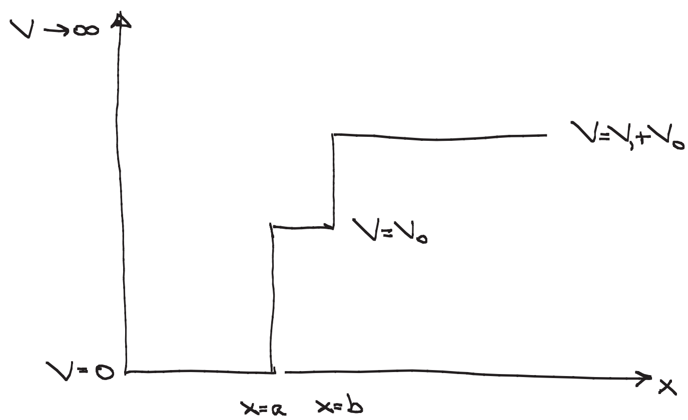
$$\frac{k\hbar^2}{2m\alpha} = \frac{\sin(ka/3)}{1 - 2\cos(ka/3)} \; .$$

(d) (15 points) *Explain* how to use the graphs on the page attached at the end to determine k and B. You should be sure that your conclusions from this graph are consistent with what you said in part (a).

Problem 2 (35 points) A particle of mass m moves in the piecewise-constant potential

$$V(x) = \begin{cases} \infty \; , & x < 0, \\ 0 \; , & 0 < x < a, \\ V_0 \; , & a < x < b, \\ V_1 + V_0 \; , & x > b. \end{cases}$$

This potential is show below. Assume that $\sqrt{2mV_0}/\hbar \gg 1$ and $\sqrt{2mV_1}/\hbar \gg 1$.



(a) (20 points) Using phase-space considerations, *estimate* the number of bound states with energy less than or equal to E, where $0 \le E \le V_0 + V_1$.

(b) (15 points) Use the result of part (a) to estimate the bound-state energies E_n as a function of an integer quantum number $n = 1, 2, \ldots$. For convenience, you can assume that $N = 2a\sqrt{2mV_0}/h = a\sqrt{2mV_0}/\pi\hbar \gg 1$ is a large integer. Don't get hung up on making a precise correspondence between energy E and integer n; the approximation isn't good enough to support such precision.

The solid (blue) line is the function

$$\frac{\sin u}{1 - 2\cos u}\ ,$$

and the dashed (purple) line is the function

$$\frac{1}{2\sin(u/2)}\ .$$