Phys 521 Quantum Mechanics I **Fall 2011**

Exam #2 **Due Thursday, October 20**
(16% of course grade) **(at lecture)**

This is an examination for Phys 521. Your score will determine 16% of your grade for the course.

This is a take-home, open-book exam. You may use the textbook, your own notes, your own homework assignments, all class handouts, including solution sets for homework assignments, but do not consult books other than our primary textbook. You may use as much time as you want to complete the exam, as long as it is turned in by the deadline. Your completed exam must be solely your own work; do not consult anyone else in doing the exam.

Do your work on sheets of paper separate from the exam. The solution you hand in for a problem should be neat and legible and in a logical order. It should represent your best effort at solving the problem and should not include false starts and detours that led nowhere.

Staple your solution(s) to this page, and label this page with your *printed* name. Then *sign and date the pledge below,* and turn in the exam as instructed. You may keep the exam question(s).

I have obeyed the rules in taking this exam. In particular, I have not consulted anyone about the exam, and the completed exam is solely my own work.

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Signature Date

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Exam #2 **(100 points)**

Problem 1 (55 points) Let the vectors $|u_1\rangle$, $|u_2\rangle$, and $|u_3\rangle$ be orthonormal basis vectors in the three-dimensional Hilbert space of some quantum system. In this problem we are concerned with the following three observables:

$$\hat{A} = |u_1\rangle\langle u_1| - |u_3\rangle\langle u_3| \; ,$$
$$\hat{B} = |u_1\rangle\langle u_3| + |u_2\rangle\langle u_2| + |u_3\rangle\langle u_1| \; ,$$
$$\hat{C} = \frac{1}{2}|u_1\rangle\langle u_3| - \frac{1}{2}|u_2\rangle\langle u_2| + \frac{1}{2}|u_3\rangle\langle u_1| \; .$$

The state of the system at $t = 0$ is given by

$$|\psi(0)\rangle = \frac{1}{2}|u_1\rangle + \frac{1}{\sqrt{2}}|u_2\rangle + \frac{1}{2}|u_3\rangle \; .$$

(a) (15 points) For each of the three operators, *give* the eigenvectors and the corresponding eigenvalues. *Identify* two complete sets of commuting observables that have *different* sets of eigenvectors.

(b) (10 points) For each of the three operators, *give* the possible results of a measurement of the operator at $t = 0$ and the probabilities for the various results.

(c) (15 points) Suppose the Hamiltonian of the system is $\hat{H} = \hbar\omega\hat{A}$. *Calculate* the state $|\psi(t)\rangle$ of the system at an arbitrary time t. For each of the three operators, *give* the possible results of a measurement of the operator at time t and the probabilities for the various results.

(d) (15 points) Suppose the Hamiltonian of the system is $\hat{H} = \hbar\omega\hat{C}$. *Calculate* the state $|\psi(t)\rangle$ of the system at an arbitrary time t. For each of the three operators, *give* the possible results of a measurement of the operator at time t and the probabilities for the various results.

Problem 2 (45 points) Consider a free particle of mass m that has position operator $\hat{x}$ and momentum operator $\hat{p}$. The particle's Hamiltonian is $\hat{H} = \hat{p}^2/2m$. At $t = 0$ the particle's (normalized) wave function is

$$\langle x|\psi(0)\rangle = \psi(x,0) = \sqrt{2}a^{3/2}xe^{-a|x|} \; ,$$

where $a > 0$.

(a) (20 points) *Derive* the initial momentum-space wave function $\langle p|\psi(0)\rangle = \bar{\psi}(p,0)$.

(b) (25 points) Using any technique at your disposal, *calculate* at $t = 0$ the following expectation values: $\langle\hat{x}\rangle_0$, $\langle\hat{p}\rangle_0$, $\langle\hat{x}^2\rangle_0$, $\langle\hat{p}^2\rangle_0$, and $\langle\hat{x}\hat{p} + \hat{p}\hat{x}\rangle_0$. *Derive* the uncertainties Δx and Δp.

You might find the following integrals useful:

$$\int_0^\infty du\, u^n e^{-u} = n!$$

$$\int_{-\infty}^{\infty} du\, \frac{u^2}{(1+u^2)^4} = \frac{\pi}{16} \qquad \int_{-\infty}^{\infty} du\, \frac{u^4}{(1+u^2)^4} = \frac{\pi}{16} \qquad \int_{-\infty}^{\infty} du\, \frac{u^4}{(1+u^2)^5} = \frac{3\pi}{128}$$