Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #1 **Due Thursday, September 1**

(30 points) **(at lecture)**

1.1 (10 points) Using the uncertainty relation, *estimate* the ground-state energy of a particle of mass m in a three-dimensional central potential well

$$V(r) = -\frac{\alpha}{r^s} ,$$

where α and s are arbitrary positive real numbers. You should assume that the ground state has zero angular momentum. Distinguish carefully the cases $s < 2$ and $s \geq 2$.

1.2 (10 points) A particle of mass m is confined to an infinite square well of width b, which has a δ function well or barrier in the middle. In symbols the potential is

$$V(x) = \begin{cases} \alpha\delta(x) , & |x| < b/2, \\ \infty , & |x| > b/2, \end{cases}$$

with $\alpha < 0$ for a well and $\alpha > 0$ for a barrier. Throughout this problem you don't need to normalize wave functions.

(a) *Find* the energy of the *odd* bound states, and *make* a rough plot of the first odd wave function $\varphi(x)$ for a typical value of α. (Hint: If this seems hard, you're on the wrong track.)

(b) For a barrier ($\alpha > 0$), *derive* a transcendental equation that determines the energies of the *even* bound states. *Indicate* how to obtain the energies of the even states by a graphical method. *Discuss* how the wave function and energy of the lowest even state change as α increases from 0 to ∞. [Hint: Assume a wave function of the form $\varphi(x) = A\sin(k|x| - \phi)$].

(c) For a well ($\alpha < 0$), *derive* a transcendental equation that determines the energies of the *even* bound states. *Indicate* how to obtain the energies of the even states by a graphical method. You will have to consider carefully what happens to the lowest-energy even state when its energy goes to zero. *Discuss* how the wave function and energy of the lowest two even states change as α decreases from 0 to $-\infty$.

1.3 (10 points) Challenge problem