Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #1 **Due Thursday, September 1**
(30 points) **(at lecture)**

1.3 (10 points) Challenge problem. **Bound state of a particle in a δ-function potential.** Consider a particle moving in one spatial dimension, whose Hamiltonian is

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x) \ ,$$

where α is a positive constant with dimensions of energy-length.

(a) *Integrate* the eigenvalue equation of H between $-\epsilon$ and $+\epsilon$. Letting ϵ approach 0, *show* that the derivative of the eigenfunction $\varphi(x)$ has a discontinuity at $x = 0$, and *determine* this discontinuity in terms of α, m, and $\varphi(0)$.

(b) Assume that the energy E of the particle is negative (i.e., we are looking for bound states). *Find* the possible energy eigenvalues of the bound states and the corresponding bound-state eigenfunctions.

(c) *Trace* the bound-state wave functions graphically, and *give* an order-of-magnitude estimate of their width Δx.

(d) *Find* the probability $\overline{d\mathcal{P}}(p) = \overline{\mathcal{P}}(p)dp$ that a measurement of the momentum of the particle in one of the normalized bound states gives a result between p and $p + dp$. For what value of p is this probability maximum? *Give* an order-of-magnitude estimate of the width Δp of the momentum distribution. Combining this result with that of part (c), *estimate* the uncertainty product $\Delta x \Delta p$.