Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #2 **Due Tuesday, September 13**
(50 points) **(at lecture)**

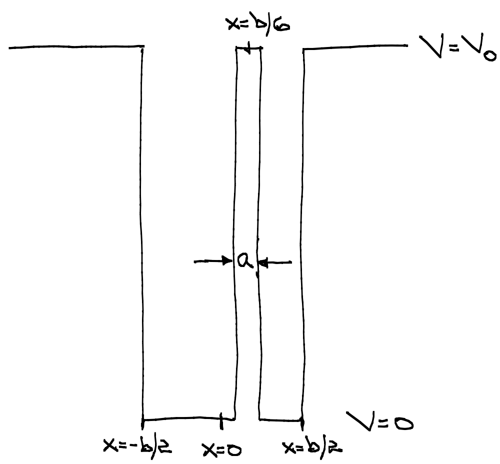
2.1 (10 points) A particle of mass m moves in the potential $V(x)$ shown below. The potential is a square well of depth V_0 and width b, which is interrupted by a barrier of height V_0 and width a, centered at $x_0 = b/6$. An explicit description of the potential is

$$V(x) = \begin{cases} V_0 \;, & x < -b/2,\ x > b/2, \text{ and } x_0 - a/2 < x < x_0 + a/2, \\ 0 \;, & \text{otherwise.} \end{cases}$$

Throughout this problem you should assume that the well is very deep, i.e.,

$$\frac{\sqrt{2mV_0}}{\hbar} b \gg 1 \;,$$

and that $a \leq b/2$.



(a) *Estimate* the number of bound states that are confined within the potential $V(x)$.

(b) Assume that the barrier is very narrow, narrow enough that

$$\frac{2mV_0 ab}{\hbar} = \left(\frac{\sqrt{2mV_0}}{\hbar} a\right)\left(\frac{\sqrt{2mV_0}}{\hbar} b\right) \ll 1 \;,$$

give approximate values for the energies of the lowest five bound states. *Justify* the approximation you make. (Hint: If you find yourself doing much work to answer this part, you are on the wrong track.)

(c) Assuming that $a = b/3$, *give* approximate values for the energies of the lowest five bound states. *Justify* your answer in words. (Hint: If you find yourself doing any work to answer this part, you are on the wrong track.)

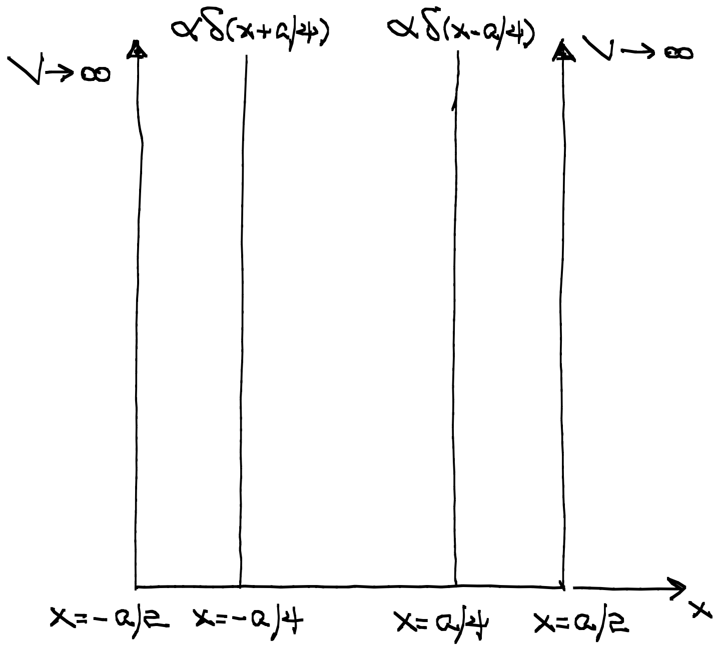
2.2 (10 points) Consider an infinite square well of width a, as shown below, which has equal-strength δ-function barriers at $x = \pm a/4$. The potential can be written as

$$V(x) = \begin{cases} \alpha\delta(x + a/4) + \alpha\delta(x - a/4)\ , & |x| < a/2, \\ \infty\ , & |x| > a/2, \end{cases}$$

with $\alpha \geq 0$. For $|x| < a/2$, the energy eigenfunctions satisfy the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + \alpha\delta(x + a/4)\varphi(x) + \alpha\delta(x - a/4)\varphi(x) = E\varphi(x)\ .$$

Throughout this problem, you do not need to normalize the wave function.



(a) Without actually solving the problem, *plot* and *discuss* how the ground-state wave function changes as α increases from 0 to ∞.

(b) *Find* the ground-state wave function $\varphi(x)$ and the ground energy E as functions of α. Your answer should be in terms of a graph that allows you to determine the parameters of the wave function and the energy $E = \hbar^2k^2/2m$ as functions of α. [Hint: The ground state has an even wave function, which can be chosen to be

$$\varphi(x) = \begin{cases} \cos kx\ , & |x| < a/4, \\ B\cos(k|x| - \phi)\ , & a/4 < |x| < a/2. \end{cases}$$

The overall normalization has been chosen to make the constant in front of $\cos kx$ equal to 1. Your graphical method should determine k, ϕ, and B.]

(c) *Check* that the ground-state wave function you found in part (b) has the behavior you guessed in part (a); you should go back and fix part (a) if you didn't guess correctly.

2.3 (10 points) CT K_I.3

2.4 (10 points) Using the WKB method, *find* approximate values for the energies of the two lowest bound states of a particle of mass m moving in a one-dimensional potential $V(x) = C|x|$, where $C > 0$.

2.5 (10 points) Challenge problem