Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #3 **Due Tuesday, October 4**
(70 points) **(at lecture)**

3.4 (10 points) Challenge problem (a). Let the vectors $|u_1\rangle$, $|u_2\rangle$, $|u_3\rangle$, and $|u_4\rangle$ be orthonormal basis vectors in the four-dimensional Hilbert space of some quantum system. In this problem we are concerned with the following four observables:

$$\hat{A} = \frac{1}{2}\Big(|u_1\rangle\langle u_1| + |u_2\rangle\langle u_2| - |u_3\rangle\langle u_3| - |u_4\rangle\langle u_4|\Big) ,$$
$$\hat{B} = \frac{1}{2}\Big(|u_1\rangle\langle u_1| - |u_2\rangle\langle u_2| + |u_3\rangle\langle u_3| - |u_4\rangle\langle u_4|\Big) ,$$
$$\hat{C} = \hat{A} + \hat{B} = |u_1\rangle\langle u_1| - |u_4\rangle\langle u_4| ,$$
$$\hat{D} = |u_1\rangle\langle u_1| + |u_2\rangle\langle u_3| + |u_3\rangle\langle u_2| + |u_4\rangle\langle u_4| .$$

The state of the system at $t = 0$ is given by

$$|\psi(0)\rangle = \frac{1}{2}\Big(|u_1\rangle + |u_2\rangle + |u_3\rangle + |u_4\rangle\Big) .$$

(a) For each of the four operators, *give* eigenvectors and the corresponding eigenvalues. *Identify* two complete sets of commuting observables that have *different* sets of eigenvectors.

(b) For each of the four operators, *give* the possible results of a measurement of the operator at $t = 0$ and the probabilities for the various results.

(c) Suppose the Hamiltonian of the system is $\hat{H} = \hbar\omega\hat{C}$. *Calculate* the state $|\psi(t)\rangle$ of the system at an arbitrary time t. If a measurement of $\hat{C}$ is made at time t, what are the possible results of the measurement and their probabilities? If a measurement of $\hat{D}$ is made at time t, what are the possible results of the measurement and their probabilities?

(d) Suppose the Hamiltonian of the system is $\hat{H} = \hbar\omega\hat{A}$. *Calculate* the state $|\psi(t)\rangle$ of the system at an arbitrary time t. If a measurement of $\hat{C}$ is made at time t, what are the possible results of the measurement and their probabilities? If a measurement of $\hat{D}$ is made at time t, what are the possible results of the measurement and their probabilities?