Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #4 **Due Tuesday, October 11**

(60 points) **(at lecture)**

4.1 (10 points) CT $\mathrm{L_{III}}$.4

4.2 (10 points) CT $\mathrm{L_{III}}$.5

4.3 (10 points) CT $\mathrm{L_{III}}$.10(a)

4.4 (10 points) CT $\mathrm{L_{III}}$.14

4.5 (10 points) Consider a quantum system with Hamiltonian $\hat{H}$ and corresponding evolution operator $\hat{U}(t,0) = e^{-(i/\hbar)\hat{H}t}$. Let $\hat{A}_S$ be a discrete nondegenerate observable with eigenvalues and Schrödinger-picture eigenvectors given by $\hat{A}_S|A_k\rangle = A_k|A_k\rangle$. The corresponding Heisenberg-picture operator is $\hat{A}_H(t) = \hat{U}^\dagger(t,0)\hat{A}_S\hat{U}(t,0)$.

(a) Show that the time-dependent state vector $|A_k, t\rangle \equiv \hat{U}^\dagger(t,0)|A_k\rangle$ is an eigenstate of $\hat{A}_H(t)$ with eigenvalue A_k.

(b) The probability to find eigenvalue A_k at time t is given in the Schrödinger picture by

$$p(A_k, t) = |\langle A_k|\psi_S(t)\rangle|^2 = |\langle A_k|\hat{U}(t,0)|\psi_S(0)\rangle|^2.$$

Write $p(A_k, t)$ in terms of Heisenberg-picture quantities, and show that it can be put in the form

$$p(A_k, t) = \langle \psi_H|\hat{P}_k(t)|\psi_H\rangle,$$

where

$$\hat{P}_k(t) = |A_k, t\rangle\langle A_k, t| = \hat{U}^\dagger(t,0)|A_k\rangle\langle A_k|\hat{U}(t,0)$$

is a Heisenberg-picture projection operator.

(c) Generalize the result of part (b) to a discrete, but degenerate observable.

4.6 (10 points) Challenge problem