Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #4 **Due Tuesday, October 11**
(60 points) **(at lecture)**

4.6 (10 points) Challenge problem. Consider a free particle of mass m that has position operator $\hat{x}$ and momentum operator $\hat{p}$. The particle's Hamiltonian is $\hat{H} = \hat{p}^2/2m$. At $t = 0$ the particle's (normalized) wave function is

$$\langle x|\psi(0)\rangle = \psi(x, 0) = \sqrt{a}e^{-a|x|} ,$$

where $a > 0$.

(a) *Derive* the initial momentum-space wave function $\langle p|\psi(0)\rangle = \bar{\psi}(p, 0)$.

(b) Using any technique at your disposal, *calculate* at $t = 0$ the following expectation values: $\langle\hat{x}\rangle_0$, $\langle\hat{p}\rangle_0$, $\langle\hat{x}^2\rangle_0$, $\langle\hat{p}^2\rangle_0$, and $\langle\hat{x}\hat{p} + \hat{p}\hat{x}\rangle_0$. *Derive* the uncertainties Δx and Δp.

(c) Using any technique at your disposal, *calculate* at time t the following expectation values: $\langle\hat{x}\rangle_t$, $\langle\hat{p}\rangle_t$, $\langle\hat{x}^2\rangle_t$, $\langle\hat{p}^2\rangle_t$, and $\langle\hat{x}\hat{p} + \hat{p}\hat{x}\rangle_t$.

You might find the following integrals useful:

$$\int_0^\infty du\, u^n e^{-u} = n! \qquad\qquad \int_{-\infty}^{\infty} du\, \frac{u^2}{(1 + u^2)^2} = \frac{\pi}{2}$$