Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #5 **Due Tuesday, November 1**
(40 points) **(at lecture)**

5.4 (10 points) Challenge problem (b). **Classical generalized force on a harmonic oscillator. II.** Consider a harmonic oscillator that is subjected to a classical generalized force $f(t)$, so that its Hamiltonian becomes

$$\hat{H}(t) = \hat{H}_0 + i\hbar[\hat{a}^\dagger f(t) - \hat{a} f^*(t)] \ ,$$

where $\hat{H}_0 = \hbar\omega\hat{a}^\dagger\hat{a}$ is the oscillator's free Hamiltonian (the zero-point energy has been omitted since it has no effect on the dynamics).

(a) *Derive* the Heisenberg equations of motion for position $\hat{x}$ and momentum $\hat{p}$ and for the creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$. *Explain* why $f(t)$ is called a generalized force, instead of just a force.

(b) *Solve* the Heisenberg equations of motion for the creation and annihilation operators, with initial conditions at $t = 0$. Your answer should involve the oscillator complex amplitude,

$$\alpha(t) = \int_0^t dt'\, f(t')e^{i\omega t'} \ ,$$

and the phase

$$\delta(t) = \tfrac{1}{2}i \int_0^t dt'\, (\alpha^*\dot{\alpha} - \alpha\dot{\alpha}^*) \ .$$

(c) Suppose that the oscillator is initially in the ground state, i.e., $|\psi(0)\rangle = |\varphi_0\rangle$. At time t, *find* the expectation values of $\hat{a}$, $\hat{a}^\dagger$, $\hat{a}^\dagger\hat{a}$, and $\hat{a}^2$ in terms of $\alpha(t)$ and $\delta(t)$. Use these expectation values to *find* the expectation values and variances of $\hat{x}$ and $\hat{p}$.