Phys 522 Quantum Mechanics II **Spring 2010**

Exam #1 **Due Monday, February 15**
(20% of course grade) **(at noon in TA's mailbox)**

This is an examination for Phys 522. Your score will determine 20% of your grade for the course.

This is a take-home, open-book exam. You may use the textbook, your own notes, your own homework assignments, all class handouts, including solution sets for homework assignments, but do not consult books other than our primary textbook. You may use as much time as you want to complete the exam, as long as it is turned in by the deadline. Your completed exam must be solely your own work; do not consult anyone else in doing the exam.

Do your work on sheets of paper separate from the exam. The solution you hand in for a problem should be neat and legible and in a logical order. It should represent your best effort at solving the problem and should not include false starts and detours that led nowhere.

Staple your solution(s) to this page, and label this page with your name. Then *sign and date the pledge below,* and turn in the exam to the TA's mailbox. You may keep the exam question(s).

I have obeyed the rules in taking this exam. In particular, I have not consulted anyone about the exam, and the completed exam is solely my own work.

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Signature Date

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Exam #1 **(100 points)**

Problem 1 (45 points) Consider two systems, the first with angular momentum $j_1 = j$ and the second with angular momentum $j_2 = j - \frac{1}{2}$. *Find* the total-angular-momentum state $|J = \frac{1}{2}, M = \frac{1}{2}\rangle$ in terms of the states $|j, j - \frac{1}{2}, m_1 m_2\rangle$.

Problem 2 (55 points) Consider N spin-1/2 particles, each with S_z eigenstates $|\frac{1}{2}, m = \frac{1}{2}\epsilon\rangle \equiv |\epsilon\rangle$. The total angular momentum is

$$\mathbf{J} = \sum_{l=1}^{N} \mathbf{S}_l = \frac{1}{2}\hbar \sum_{l=1}^{N} \boldsymbol{\sigma}_l \ .$$

There is one subspace of (maximal) total angular momentum $J = N/2$; the J_z eigenstates in this subspace are given by

$$|JM\rangle = \sqrt{\frac{(J+M)!(J-M)!}{(2J)!}} \sum_{\substack{\epsilon_1,\ldots\epsilon_N \\ M=(n_+-n_-)/2}} |\epsilon_1, \ldots, \epsilon_N\rangle \ ,$$

where the sum is restricted to those strings $\epsilon_1, \ldots, \epsilon_N$ that have

$$\frac{1}{2}\sum_{l=1}^{N} \epsilon_l = \frac{1}{2}(n_+ - n_-) = M \ .$$

In such a string, the number of particles with spin up is $n_+ = N/2 + M = J + M$, and the number of particles with spin down is $n_- = N/2 - M = J - M$.

(a) (15 points) Suppose the particles are in the state $|JM\rangle$ and that a measurement of S_z is performed on the first particle. What is the probability for finding spin up? How would this answer change if the measurement of S_z were performed on the lth particle?

(b) (10 points) *Explain* why there are $2J - 1 = N - 1$ subspaces of total angular momentum $J - 1 = N/2 - 1$.

(c) (15 points) Let's concentrate here on the particular subspace of total angular momentum $J - 1$ that arises in the following way: start with $N - 1$ particles in the symmetric subspace of (maximal) total angular momentum $J - \frac{1}{2}$; then add the final (Nth) particle to get two angular-momentum subspaces, one the symmetric subspace of (maximal) angular momentum J and the other of angular momentum $J - 1$. Let's denote the J_z eigenstates in the latter subspace by $|J - 1, M\rangle_N$, where the subscript N means that we added the Nth particle to the symmetric space of the previous $N - 1$ particles. Use the Clebsch-Gordon coefficients below to *find* the states $|J - 1, M\rangle_N$ in terms of the product states $|\epsilon_1, \ldots, \epsilon_N\rangle$.

(d) (15 points) Suppose the particles are in the state $|J - 1, M\rangle_N$ of part (c) and that a measurement of S_z is performed on the Nth particle. What is the probability for finding spin up? What is the probability for finding spin up if the measurement is instead performed on the first particle?

Clebsch-Gordon coefficients for adding spin j and spin $\frac{1}{2}$

$$\langle j\tfrac{1}{2}, m \pm \tfrac{1}{2}, \mp\tfrac{1}{2} | j + \tfrac{1}{2}, m\rangle = \sqrt{\frac{j \mp m + \frac{1}{2}}{2j+1}}$$

$$\langle j\tfrac{1}{2}, m \pm \tfrac{1}{2}, \mp\tfrac{1}{2} | j - \tfrac{1}{2}, m\rangle = \pm\sqrt{\frac{j \pm m + \frac{1}{2}}{2j+1}}$$