**Phys 522 Quantum Mechanics II** **Spring 2010**

**Homework Assignment #2** **Due Thursday, February 11**
**(60 points)** **(at lecture)**

2.1 (10 points) CT $\mathrm{G_X}$.3

2.2 (10 points) CT $\mathrm{G_X}$.5

2.3 (10 points) Consider two systems, the first of angular momentum $j_1 = j$ and the second of angular momentum $j_2 = 1$ or $j_2 = 2$.

(a) For $j_2 = 1$, *find* the states $|J = j+1, M = j\rangle$ and $|J = j, M = j\rangle$.

(b) For $j_2 = 2$, *find* the states $|J = j, M = j\rangle$.

2.4 (10 points) In this problem, we consider the rotation $\mathcal{R}_{\mathbf{e}_y}(\pi)$ that rotates by $\pi$ about the $y$ axis.

(a) Use the fact that $\mathbf{J}$ is a vector operator to *show* that $|jm\rangle$ rotates to a phase times $|j,-m\rangle$, i.e., $R_{\mathbf{e}_y}(\pi)|jm\rangle = e^{i\delta_{jm}}|j,-m\rangle$.

(b) *Find* the phase $\delta_{jm}$ and thus *find* the rotation matrix $D^{(j)}_{m'm}(\mathcal{R})$ for this rotation. (Hint: Consider the angular-momentum states $|jm\rangle$ as making up the space of maximum total angular momentum for $2j$ spin-1/2 particles.)

(c) Use these results to *verify* how the the Clebsch-Gordon coefficients change under a change of sign of the magnetic quantum numbers.

2.5 (10 points) The rotation matrices for angular momentum $j$ are defined by

$$D^{(j)}_{m'm}(\mathcal{R}) \equiv \langle jm'|e^{-i\alpha\mathbf{u}\cdot\mathbf{J}/\hbar}|jm\rangle \ .$$

(a) *Write* the explicit form of the rotation matrices when $\alpha = d\alpha$ is infinitesimal.

For the remainder of the problem, we specialize to $j = 1$. The matrices $D^{(1)}$ are a three-dimensional representation of the rotation group, so there must be another orthonormal basis in which the matrix representation is the standard one that comes from considering rotations in three real spatial dimensions. We're going to show that the desired orthonormal basis is

$$\begin{aligned}
|1\rangle &= \frac{1}{\sqrt{2}}\big(|j=1,m=+1\rangle - |j=1,m=-1\rangle\big) \ ,\\
|2\rangle &= -\frac{i}{\sqrt{2}}\big(|j=1,m=+1\rangle + |j=1,m=-1\rangle\big) \ ,\\
|3\rangle &= -|j=1,m=0\rangle \ .
\end{aligned}$$

We let Latin letters at the beginning of the alphabet take on the values 1, 2, 3.

(b) *Show* that the matrix elements $D_{ab} \equiv \langle a|e^{-i\,d\alpha\,\mathbf{u}\cdot\mathbf{J}/\hbar}|b\rangle$ have the standard form for an infinitesimal rotation in three dimensions.

Since you know the form of $D_{ab}$ for a rotation by a finite angle, we could transform back to the $|1m\rangle$ basis to find the matrix $D^{(1)}_{m'm}$ for finite rotations. The result would agree with that found in 2.5(d), but demonstrating the agreement probably isn't worth the effort.

2.6 (10 points) Challenge problem.