4.2 (10 points) Challenge problem. A hydrogen atom is placed in a magnetic field $\mathbf{B} = B_0 \mathbf{e}_z$.

(a) Estimate the magnetic-field magnitude such that the Zeeman shifts due to the magnetic field are comparable to the fine-structure energy shifts.

(b) Considering both the Zeeman interaction and the fine-structure effects, work out in detail for $n = 1$ and $n = 2$ the first-order energy shifts and the zero-order energy eigenstates for arbitrary field strength B_0. Make a rough plot of the energies as a function of B_0.

(c) Specialize the results of part (b) to weak and strong fields, and discuss the behavior of the energy shifts and the energy eigenstates in these two limits.