Phys 571 Quantum Computation **Fall 2009**

Homework Problem 1.1 **Due Thursday, September 3**
(10 points) **(at lecture)**

1.1 **Two-bit classical gates.**

(a) *Show* that the most general two-bit classical gate can be written as

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \oplus M \begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} p \\ q \end{pmatrix} xy \ ,$$

where a, b, p, q, and the four entries in the 2×2 matrix M can be either 0 or 1.

(b) *Show* that the most general *reversible* two-bit classical gate can be written as

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \oplus M \begin{pmatrix} x \\ y \end{pmatrix} \ ,$$

where a and b can be either 0 or 1, and where M can be any of the following matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} , \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} , \quad \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} , \quad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} .$$

(c) The constants a and b leave the output unchanged or do a NOT on the output for the two bits. Since we can easily understand the effect of a and b, we now set them to zero. In this situation, *characterize* the action of the six reversible M matrices of part (b).