1.6 **Two-level transitions.**

(a) The FREDKIN gate is a controlled SWAP. By writing SWAP as a product of three controlled-NOTs, *write* FREDKIN, with the control on the third qubit, as a product of three TOFFOLIs.

We showed in class that the three-qubit two-level transition that exchanges $|000\rangle$ and $|111\rangle$, leaving all other computational basis states unchanged, is effected by the circuit below.
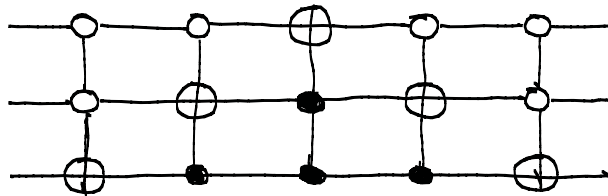
(b) By converting all the controls-on-zero to controls-on-one, *convert* this circuit to an equivalent one that consists of TOFFOLIs interspersed with spin flips. You should recognize a FREDKIN gate in the middle of your new circuit.

(c) By following the evolution of input state $|a, b, c\rangle$ through the circuit of part (b), *show* explicitly that the circuit effects the desired two-level transition. (Hint: I found it useful to introduce the "majority-vote" Boolean function $q = ab \oplus ac \oplus bc$, which takes inputs with no 1s or one 1 to 0 and inputs with two 1s or three 1s to 1.)