**Phys 571 Quantum Computation** **Fall 2009**

**Homework Problem 2.1** **Due Thursday, September 24**
**(10 points)** **(at lecture)**

2.1 **Transformation of Pauli products by C-NOT and C-SIGN.** Let $C$ denote either the controlled-NOT gate or the controlled-SIGN gate, with the first qubit as control and the second qubit as target. In this problem we investigate how C-NOT and C-SIGN transform Pauli products.

(a) Use circuit identities to *derive* the following transformations for $C =$ C-NOT:

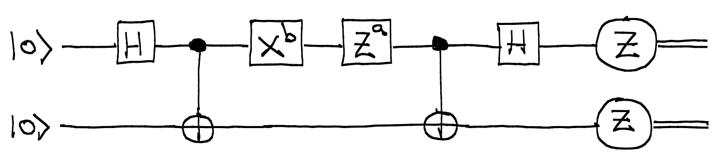
$$\begin{aligned} C(X \otimes I)C^\dagger &= X \otimes X \\ C(I \otimes X)C^\dagger &= I \otimes X \\ C(Z \otimes I)C^\dagger &= Z \otimes I \\ C(I \otimes Z)C^\dagger &= Z \otimes Z\ . \end{aligned}$$

(b) Use the transformations of part (a) to *derive* how the C-NOT transforms all Pauli products.

(c) *Repeat* parts (a) and (b) for the C-SIGN gate.

Notice that both C-NOT and C-SIGN take Pauli products to Pauli products.

(d) The circuit for superdense coding is shown below. The initial Hadamard–C-NOT combination creates the Bell entangled state $|\beta_{00}\rangle$. The first qubit is distributed to Alice, and the second is given to Bob. Alice encodes two bits, $a$ and $b$, by applying $Z^aX^b$ to her qubit, yielding the Bell state $|\beta_{ab}\rangle$, and then sends her qubit to Bob. The C-NOT–Hadamard followed by measurement in the standard basis is equivalent to Bob's measuring the two qubits in the Bell basis; thus Bob reads out the encoded data.




Use the transformation properties of the C-NOT to transform this circuit to an equivalent form for which it is obvious that the measurements yield $a$ for the top qubit and $b$ for the bottom qubit.