Phys 571 Quantum Computation **Fall 2009**

Homework Problem 2.2 **Due Thursday, September 24**
(10 points) **(at lecture)**

2.2 **Parity and phase observables for Bell states.** The four orthogonal Bell states for two qubits are defined as

$$|\beta_{ab}\rangle = \frac{1}{\sqrt{2}}(|0b\rangle + (-1)^a|1\bar{b}\rangle) = \frac{1}{\sqrt{2}}\sum_c (-1)^{ac}|c, c \oplus b\rangle$$

where a is called the phase bit, because it determines the relative phase in the superposition, and b is called the parity bit, because it determines the parity of the two states in superposition.

(a) *Show* that the Bell-state projection operators are given by

$$|\beta_{ab}\rangle\langle\beta_{ab}| = \frac{1}{4}\Big(I \otimes I + (-1)^b Z \otimes Z + (-1)^a X \otimes X - (-1)^{a+b} Y \otimes Y\Big) .$$

(b) Invert these relations to *show* that the (commuting) Pauli products on the right side of the expression in (a) are given in terms of Bell-state projectors by

$$\begin{aligned}
I \otimes I &= +|\beta_{00}\rangle\langle\beta_{00}| + |\beta_{10}\rangle\langle\beta_{10}| + |\beta_{01}\rangle\langle\beta_{01}| + |\beta_{11}\rangle\langle\beta_{11}| \, , \\
Z \otimes Z &= +|\beta_{00}\rangle\langle\beta_{00}| + |\beta_{10}\rangle\langle\beta_{10}| - |\beta_{01}\rangle\langle\beta_{01}| - |\beta_{11}\rangle\langle\beta_{11}| \, , \\
X \otimes X &= +|\beta_{00}\rangle\langle\beta_{00}| - |\beta_{10}\rangle\langle\beta_{10}| + |\beta_{01}\rangle\langle\beta_{01}| - |\beta_{11}\rangle\langle\beta_{11}| \, , \\
Y \otimes Y &= -|\beta_{00}\rangle\langle\beta_{00}| + |\beta_{10}\rangle\langle\beta_{10}| + |\beta_{01}\rangle\langle\beta_{01}| - |\beta_{11}\rangle\langle\beta_{11}| \, ,
\end{aligned}$$

Thus these Pauli products are simultaneously diagonal in the Bell basis.

(c) *Explain* how measurements of the Pauli products, $Z \otimes Z$ and $X \otimes X$, determine the parity and phase of a Bell state.