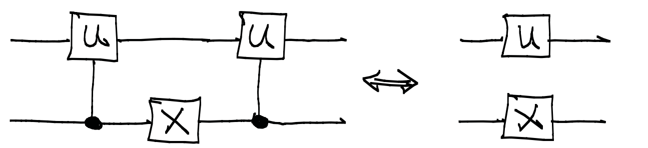
Phys 571 Quantum Computation **Fall 2009**

Homework Problem 2.4 **Due Thursday, October 1**
(10 points) **(at lecture)**

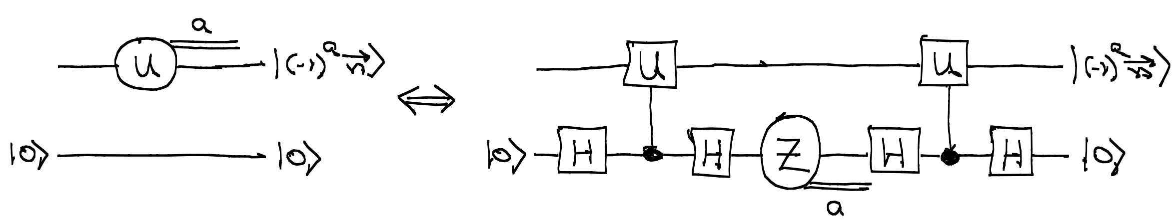
2.4 **Measurement models.** When measuring an observable, it is often desirable not to measure the observable directly, but rather to couple the system to an ancilla in such a way that the information about the observable is transferred to the ancilla and then to make a measurement on the ancilla. Such a procedure is called a *measurement model.* In this problem we derive the canonical measurement model for a qubit.

Throughout this problem, $U = \boldsymbol{\sigma} \cdot \mathbf{n}$ is both unitary ($UU^\dagger = I$) and Hermitian ($U = U^\dagger$). It thus has eigenvalues ± 1 and can be either a gate or a measured observable.

(a) *Show* the circuit identity below.



(b) Use the circuit identity of part (a) to *demonstrate* the measurement model below. (Hint: As a first step, extend the measurement to a partial measurement $U \otimes Z$.) Notice that if one is not interested in the post-measurement states of the qubits, one can omit the gates after the measurement of Z in the right-hand circuit.



(c) *Verify* that the circuit on the right in part (b) works as advertised by tracking the quantum state through the circuit.

(d) For the case $U = Z$, *simplify* the right-hand circuit of (b) by eliminating the Hadamard gates. The result is the canonical model for a measurement of Z.