2.4 **Measurement models.** When measuring an observable, it is often desirable not to measure the observable directly, but rather to couple the system to an ancilla in such a way that the information about the observable is transferred to the ancilla and then to make a measurement on the ancilla. Such a procedure is called a *measurement model*. In this problem we derive the canonical measurement model for a qubit.

Throughout this problem, \( U = \sigma \cdot n \) is both unitary \((UU^\dagger = I)\) and Hermitian \((U = U^\dagger)\). It thus has eigenvalues \(\pm 1\) and can be either a gate or a measured observable.

(a) Show the circuit identity below.

\[
\begin{array}{c}
\text{U} \quad \text{U} \\
\text{X} \quad \text{X}
\end{array}
\]

(b) Use the circuit identity of part (a) to *demonstrate* the measurement model below. (Hint: As a first step, extend the measurement to a partial measurement \(U \otimes Z\).) Notice that if one is not interested in the post-measurement states of the qubits, one can omit the gates after the measurement of \(Z\) in the right-hand circuit.

\[
\begin{array}{c}
\text{U} \quad \text{U} \\
\text{H} \quad \text{H} \quad \text{Z} \quad \text{H} \quad \text{H} \\
\text{H} \quad \text{H} \\
\text{H} \quad \text{H} \quad \text{H} \quad \text{H} \\
|\psi\rangle \quad |\phi\rangle \quad |\psi\rangle \quad |\phi\rangle
\end{array}
\]

(c) Verify that the circuit on the right in part (b) works as advertised by tracking the quantum state through the circuit.

(d) For the case \(U = Z\), *simplify* the right-hand circuit of (b) by eliminating the Hadamard gates. The result is the canonical model for a measurement of \(Z\).