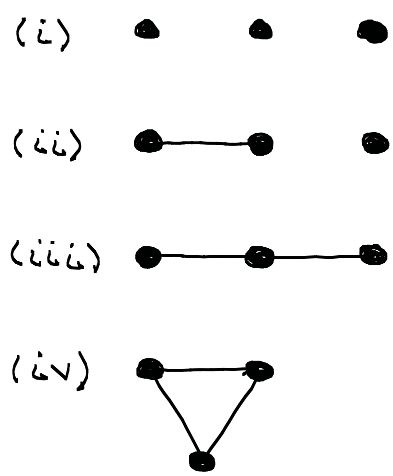
Phys 571 Quantum Computation **Fall 2009**

Homework Problem 2.6 **Due Tuesday, October 13**
(10 points) **(at lecture)**

2.6 **Three-qubit graph states.** A graph of nodes connected by edges can be associated with a quantum state called a *graph state*. Each node represents a qubit that is initially in the state $|0\rangle$; the graph state is generated by applying a Hadamard to each qubit, taking it to the state $|\bar{0}\rangle$, and then applying a C-SIGN to pairs of qubits that are connected by an edge. The C-SIGNs can be applied in any order since they commute. Cluster states are a special case of graph states for which the graph is a square lattice.

For three qubits, the possible graph states, up to SWAPs, are represented by the following four graphs: (i) three nodes with no edges; (ii) three nodes along a line with the left two nodes connected; (iii) three nodes along a line with the middle node connected to the outer two; (iv) three nodes with an edge between any two, making a triangle. These graphs are shown below.



(a) For each of the graphs above, *write* the corresponding state in the standard (Z) basis.

(b) States that are connected by SWAPs and local unitaries can be regarded as equivalent for entanglement purposes, i.e., as having the same amount of entanglement. *Show* that the four states above are all inequivalent, except perhaps states (iii) and (iv). (Hint: Consider the eigenvectors and eigenvalues of the marginal density operators.)

(c) The W state for three qubits is defined to be

$$|W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)\ .$$

Show that the W state is not equivalent to any of the three-qubit graph states above.

(d) *Show* that states (iii) and (iv) are equivalent. A smart way to do this uses a circuit identity.