

**Homework Problem 3.4**  
(10 points)**Due Thursday, November 19**  
(at lecture)

**3.4 Two-qubit quantum search and superdense coding.** In this problem we consider the two-qubit ( $N = 4$ ) version of Grover search for which there is one solution ( $M = 1$ ). In just one iteration the algorithm identifies without error which of the four possible Boolean functions (two-bit Boolean functions with one input outputting true) is applied by the oracle.

(a) *Write* the four Boolean functions as circuits involving a Toffoli gate surrounded as necessary by spin flips.

(b) *Write* the entire circuit for this case of the quantum search algorithm, and *verify* directly that the circuit identifies the Boolean function used by the oracle.

(c) The circuit of part (b), with its two controlled operations separated by operations that encode the Boolean function, is reminiscent of superdense coding. Using circuit identities, *convert* the circuit of part (b) to an equivalent circuit for superdense coding.

(d) Using further circuit identities, *eliminate* the controlled operations from the circuit of part (c), leaving an equivalent circuit that makes it obvious—and I mean *completely* obvious—that the search algorithm identifies the Boolean function used by the oracle.