Phys 571 Quantum Computation **Fall 2009**

Homework Problem 4.3 **Due Thursday, December 10**
(10 points) **(at lecture)**

4.3 **n-bit repetition code.** The n-bit repetition code has generator matrix

$$G = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} ,$$

giving code words

$$\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} .$$

The distance of the code is $d = n$, so this is a $[n, 1, n]$ code. When $n = 2t + 1$ is odd, we are guaranteed that the code can correct errors on up to $t = (n - 1)/2$ bits, and when $n = 2t + 2$ is even, the code can correct errors on up to $t = (n - 2)/2$ bits. It is possible, however, that the code can correct more errors than this.

We now assume that n is odd. The number of errors (including no error) guaranteed to be correctible is

$$\mathcal{N} = \sum_{j=0}^{t} \binom{n}{j} .$$

This is clearly half the total number of errors, 2^n, so $\mathcal{N} = 2^{n-1}$.

(a) *Find* a parity-check matrix H that involves only two-bit parity checks.

(b) *Find* the syndromes for all errors on t or fewer bits. *Discuss* how the syndromes identify the bits in error, and *show* that these errors fill up the space of syndromes.